Contents

Preface to the second edition xv
Preface to the first edition xvii
Acknowledgements (second edition) xix
Acknowledgements (first edition) xxi
Author xxiii

1 Review of some fundamentals 1

1.1 *Introduction* 1
1.2 *The role of modelling (linear and nonlinear, discrete and continuous systems, deterministic and random data)* 2
1.3 *Some definitions and methods* 4
1.3.1 *The phenomenon of beats* 7
1.3.2 *Displacement, velocity and acceleration* 9
1.3.3 *Quantification of vibration level and the decibel scale* 10
1.4 *Springs, dampers and masses* 14
1.5 *Summary and comments* 19

2 Mathematical preliminaries 21

2.1 *Introduction* 21
2.2 *Fourier series and Fourier transform* 21
2.2.1 *Periodic functions: Fourier series* 22
2.2.2 *Nonperiodic functions: Fourier transform* 29
2.2.3 *Main properties of Fourier transforms* 33
2.2.4 *Some mathematical facts* 36
2.2.5 *The bandwidth theorem (uncertainty principle)* 39

2.3 *Laplace transform* 40
2.3.1 *Laplace transforms: Basic properties and some examples* 42
2.4 *Dirac delta and related topics* 48
2.4.1 *A short introduction to distributions* 53
2.5 *The notion of Hilbert space* 62
2.5.1 *Hilbert spaces l^2 and L^2* 67
2.5.1.1 *The spaces $l^2(C)$ and $l^2(R)$* 67
2.5.1.2 *The space L^2* 68
2.5.2 *Sturm–Liouville problems* 72
2.5.3 *Some generalisations* 79

3 Analytical mechanics: An overview 83

3.1 *Introduction* 83
3.2 *Systems of material particles* 84
3.2.1 *Generalised coordinates, constraints and degrees of freedom* 85
3.3 *The principle of virtual work and d'Alembert's principle: Lagrange's and Hamilton's equations* 87
3.3.1 *Hamilton's equations* 91
3.4 *Lagrange's equations: Fundamental properties, some generalisations and complements* 93
3.4.1 *Invariance in form of LEs and monogenic forces* 93
3.4.2 *The structure of the kinetic energy and the conservation of energy* 94
3.4.3 *Elastic forces, viscous forces and Rayleigh dissipation function* 97
3.4.4 *More coordinates than degrees of freedom: Lagrange's multipliers* 99
3.5 *Hamilton's principle* 102
3.5.1 *More than one independent variable: The equation of motion of continuous systems* 107
3.5.2 *The boundary terms in Hamilton's principle: Natural boundary conditions* 109
3.6 *Small-amplitude oscillations* 112

4 Single degree of freedom systems 119

4.1 *Introduction* 119
4.2 *Harmonic oscillator I: Free vibration* 120
4.2.1 *Undamped free vibrations* 121

4.2.2 *Damped free vibration* 124
4.2.2.1 *Case 1. Critically damped motion:* $\zeta = 1$ $(c = c_{cr})$ 125
4.2.2.2 *Case 2. Overdamped motion:* $\zeta > 1$ $(c > c_{cr})$ 125
4.2.2.3 *Case 3. Underdamped motion:* $0 < \zeta < 1$ $(c < c_{cr})$ 126
4.2.3 *Logarithmic decrement* 128
4.2.4 *Further analogies* 130
4.3 *Harmonic oscillator II: Forced vibration* 131
4.3.1 *Forced vibration: Harmonic excitation* 132
4.3.2 *Force transmissibility and harmonic motion of the support* 137
4.3.3 *Resonant response of damped and undamped SDOF systems* 140
4.3.4 *Some energy considerations* 142
4.4 *Damping in real systems, equivalent viscous damping* 144
4.4.1 *Measurement of damping* 147
4.4.1.1 *Free-vibration decay* 147
4.4.1.2 *Resonant response* 148
4.4.1.3 *Half-power bandwidth* 149
4.4.1.4 *Energy loss per cycle* 149
4.4.1.5 *Frequency response function* 150
4.5 *Summary and comments* 154

5 More SDOF systems: Shock response, transient response and some approximate methods 157

5.1 *Introduction* 157
5.2 *Time domain: Impulse response function and Duhamel integral* 158
5.2.1 *Excitation due to support motion* 168
5.2.2 *Concept of shock and response spectrum* 170
5.3 *Frequency and Laplace domains: Frequency response function and transfer function* 175
5.3.1 *Response to periodic excitation* 175
5.3.2 *Fourier and Laplace transform methods* 178
5.3.3 *Relationship between the characterising functions in time, frequency and Laplace domains* 182
5.4 *Generalised SDOF systems* 184

5.5 Rayleigh (energy) method and improved Rayleigh method 191
5.6 Summary and comments 198

6 Multiple degrees of freedom (MDOF) systems 201

6.1 Introduction 201
6.2 A simple undamped 2-DOF system: Free vibration 202
6.3 Undamped n-DOF systems: Free vibration 206
6.3.1 Eigenvectors' orthogonality relations and normalisation 209
6.3.2 General solution of the undamped free-vibration problem, degeneracy and normal coordinates 213
6.3.2.1 Eigenvalue degeneracy 215
6.3.2.2 Normal coordinates 216
6.4 Eigenvalues and eigenvectors sensitivity analysis 224
6.4.1 Light damping as a perturbative term 229
6.5 A few considerations on the structure and properties of the matrices M, K and C 231
6.5.1 Mass properties 232
6.5.2 Elastic properties 234
6.5.3 More mass- and stiffness-orthogonality conditions 239
6.6 Unrestrained systems: Rigid-body modes 240
6.7 Damped systems: Proportional and nonproportional damping 246
6.7.1 Proportional damping 246
6.7.2 Nonproportional damping 250
6.8 Generalised and complex eigenvalue problems: Reduction to standard form 253
6.8.1 Undamped systems 253
6.8.2 Viscously damped systems 256
6.9 Summary and comments 260

7 More MDOF systems: Forced vibration and response analysis 263

7.1 Introduction 263
7.2 Mode superposition 264
7.2.1 Mode displacement and mode acceleration methods 269
7.3 Harmonic excitation: Proportional viscous damping 271
7.4 Time-domain and frequency-domain response 274
7.4.1 A few comments on FRFs 276
7.4.2 More on FRFs: Kramers–Kronig relations 278
7.5 Systems with rigid-body modes 281

7.6 *The case of nonproportional viscous damping 282*
7.6.1 *Harmonic excitation and receptance FRF matrix 285*
7.7 *MDOF systems with hysteretic damping 289*
7.8 *A few remarks on other solution strategies: Laplace transform and direct integration 291*
7.8.1 *Laplace transform method 292*
7.8.2 *Direct integration methods 293*
7.9 *Frequency response functions of a 2-DOF system 296*
7.10 *Summary and comments 305*

8 Continuous systems 309

8.1 *Introduction 309*
8.2 *The flexible string in transverse motion 310*
8.2.1 *The initial value problem 313*
8.2.2 *Sinusoidal waves 314*
8.2.3 *Some energy considerations 316*
8.2.4 *The presence of boundaries 318*
8.3 *Free vibration of a finite string: Standing waves and normal modes 320*
8.4 *Axial and torsional vibrations of rods 326*
8.5 *Flexural (bending) vibrations of beams 330*
8.5.1 *Case 1. Both ends simply supported (pinned–pinned configuration) 332*
8.5.2 *Case 2. One end clamped and one end free (cantilever configuration) 333*
8.5.3 *Case 3. Both ends clamped (clamped–clamped configuration) 334*
8.5.4 *Case 4. Both ends free (free–free configuration) 335*
8.5.5 *Axial force effects on flexural vibrations 336*
8.5.6 *The effects of shear deformation and rotary inertia (Timoshenko beam) 339*
8.5.6.1 *Case 1. Shear deflection alone 343*
8.5.6.2 *Case 2. Rotary inertia alone 343*
8.6 *A two-dimensional continuous system: The flexible membrane 344*
8.6.1 *The circular membrane with fixed edge 346*
8.7 *The differential eigenvalue problem 349*
8.7.1 *The differential eigenvalue problem: Some further considerations 355*

8.8 Bending vibrations of thin plates 361
8.8.1 Circular plates 364
8.8.2 Rectangular plates 367
8.9 Forced vibration and response analysis: The modal approach 372
8.9.1 Forced response of continuous systems: Some examples 378
8.10 Some final considerations: Alternative form of FRFs and the introduction of damping 388
8.11 Summary and comments 392

9 MDOF and continuous systems: Approximate methods 395

9.1 Introduction 395
9.2 The Rayleigh quotient 396
9.2.1 Courant–Fisher max–min and min–max characterisation of eigenvalues and the eigenvalue separation property 399
9.2.2 Systems with lumped masses: Dunkerley's formula 405
9.3 The Rayleigh–Ritz method 406
9.3.1 The Rayleigh–Ritz method (and the assumed-modes method) for continuous systems 410
9.3.2 Continuous systems: A few comments on admissible and comparison functions 415
9.4 Summary and comments 419

10 Experimental modal analysis 421

10.1 Introduction 421
10.2 Experimental modal analysis: Overview of the fundamentals 422
10.2.1 FRFs of SDOF systems 424
10.2.2 FRFs of MDOF systems 435
10.3 Modal testing procedures 444
10.3.1 Supporting the structure 445
10.3.2 Excitation systems 446
10.3.3 Measurement of response 449
10.3.4 Excitation functions 450
10.4 A few selected topics in experimental modal analysis 453
10.4.1 Characteristic phase lag theory and Asher's method 454

10.4.2 Single input–single output test configuration: FRF measurement and the presence of noise in the input and output signals 457
10.4.2.1 Case 1. Noise in excitation signal, no noise in response signal 460
10.4.2.2 Case 2. No noise in the excitation signal, noise in the response signal 461
10.4.2.2 Case 3. Noise in both excitation and response signals 462
10.4.3 Identification of modal parameters: Curve fitting 464
10.5 Summary and comments 473

11 Probability and statistics: Preliminaries to random vibrations 477

11.1 Introduction 477
11.2 On the concept of probability 477
11.2.1 Some heuristic considerations 479
11.2.2 A short digression on combinatorial analysis 481
11.3 Probability: Axiomatic formulation and some results 482
11.4 Random variables and distribution functions 487
11.4.1 Discrete and (absolutely) continuous random variables 488
11.4.2 Some examples 489
11.4.3 Expectation and moments of a random variable 492
11.4.4 Characteristic functions 496
11.5 Random vectors 499
11.5.1 Marginal distributions and independent random variables 501
11.5.2 Expectation, moments and characteristic functions of random vectors 503
11.5.3 Two examples: The multinomial and the multivariate normal distributions 508
11.5.4 Functions of random vectors 511
11.6 More on conditional probability 514
11.6.1 Conditional expectation 517
11.7 Convergences and the law of large numbers 519
11.7.1 The central limit theorem 521
11.8 A few remarks on probability and statistics 525

12 Stochastic processes and random vibrations 527

12.1 Introduction 527
12.2 The concept of random process 528
12.2.1 Stationary processes 532
12.2.2 Properties of correlation and covariance functions 534
12.2.3 Ergodic processes 536
12.3 Basic calculus of random processes 540
12.4 Spectral representation of random processes 544
12.4.1 Some properties of spectral densities 548
12.4.2 Narrowband and broadband processes 549
12.5 Response of linear systems to random excitation 553
12.5.1 SDOF system response to broadband excitation 558
12.5.2 Transient response 559
12.5.3 One output and more than one input 562
12.5.4 Multiple inputs and multiple outputs 566
12.5.5 Response of MDOF and continuous systems 568
12.6 Stationary narrowband processes: A few selected topics 571
12.6.1 Threshold crossing rates 572
12.6.2 Peak distribution 574
12.6.3 Notes on fatigue damage due to random excitation 577
12.7 Summary and comments 581

Appendix A 585
Appendix B 623
References 629
Index 635