

Wiley Series in Microwave and Optical Engineering • Kai Chang, Series Editor

FUNDAMENTALS OF  
**WAVELETS**

Theory, Algorithms, and  
Applications

SECOND EDITION

JAIDEVA C. GOSWAMI  
ANDREW K. CHAN

 **WILEY**

# Contents

Preface to the Second Edition	xv
Preface to the First Edition	xvii
<b>1 What Is This Book All About?</b>	<b>1</b>
<b>2 Mathematical Preliminary</b>	<b>6</b>
2.1 Linear Spaces	6
2.2 Vectors and Vector Spaces	8
2.3 Basis Functions, Orthogonality, and Biorthogonality	11
2.3.1 Example	11
2.3.2 Orthogonality and Biorthogonality	11
2.4 Local Basis and Riesz Basis	14
2.4.1 Haar Basis	15
2.4.2 Shannon Basis	15
2.5 Discrete Linear Normed Space	17
2.5.1 Example 1	17
2.5.2 Example 2	18
2.6 Approximation by Orthogonal Projection	18
2.7 Matrix Algebra and Linear Transformation	19
2.7.1 Elements of Matrix Algebra	20
2.7.2 Eigenmatrix	21
2.7.3 Linear Transformation	22
2.7.4 Change of Basis	23
2.7.5 Hermitian Matrix, Unitary Matrix, and Orthogonal Transformation	24
2.8 Digital Signals	25
2.8.1 Sampling of Signal	25
2.8.2 Linear Shift-Invariant Systems	26
2.8.3 Convolution	26
2.8.4 $z$ -Transform	28

2.8.5	Region of Convergence	29
2.8.6	Inverse $z$ -transform	31
2.9	Exercises	31
2.10	References	33
<b>3</b>	<b>Fourier Analysis</b>	<b>34</b>
3.1	Fourier Series	34
3.2	Examples	36
3.2.1	Rectified Sine Wave	36
3.2.2	Comb Function and the Fourier Series Kernel $K_N(t)$	37
3.3	Fourier Transform	39
3.4	Properties of Fourier Transform	41
3.4.1	Linearity	41
3.4.2	Time Shifting and Time Scaling	41
3.4.3	Frequency Shifting and Frequency Scaling	42
3.4.4	Moments	42
3.4.5	Convolution	43
3.4.6	Parseval's Theorem	43
3.5	Examples of Fourier Transform	44
3.5.1	The Rectangular Pulse	44
3.5.2	The Triangular Pulse	45
3.5.3	The Gaussian Function	46
3.6	Poisson's Sum and Partition of Unity	47
3.6.1	Partition of Unity	49
3.7	Sampling Theorem	51
3.8	Partial Sum and Gibb's Phenomenon	53
3.9	Fourier Analysis of Discrete-Time Signals	54
3.9.1	Discrete Fourier Basis and Discrete Fourier Series	54
3.9.2	Discrete-Time Fourier Transform (DTFT)	56
3.10	Discrete Fourier Transform (DFT)	58
3.11	Exercises	59
3.12	References	60
<b>4</b>	<b>Time-Frequency Analysis</b>	<b>61</b>
4.1	Window Function	63
4.2	Short-Time Fourier Transform	64
4.2.1	Inversion Formula	65
4.2.2	Gabor Transform	66
4.2.3	Time-Frequency Window	66
4.2.4	Properties of STFT	67
4.3	Discrete Short-Time Fourier Transform	68
4.4	Discrete Gabor Representation	70
4.5	Continuous Wavelet Transform	71
4.5.1	Inverse Wavelet Transform	73
4.5.2	Time-Frequency Window	74

4.6	Discrete Wavelet Transform	76
4.7	Wavelet Series	77
4.8	Interpretations of the Time-Frequency Plot	78
4.9	Wigner-Ville Distribution	80
4.9.1	Gaussian Modulated Chirp	81
4.9.2	Sinusoidal Modulated Chirp	82
4.9.3	Sinusoidal Signal	83
4.10	Properties of Wigner-Ville Distribution	83
4.10.1	A Real Quantity	85
4.10.2	Marginal Properties	85
4.10.3	Correlation Function	86
4.11	Quadratic Superposition Principle	86
4.12	Ambiguity Function	88
4.13	Exercises	89
4.14	Computer Programs	90
4.14.1	Short-Time Fourier Transform	90
4.14.2	Wigner-Ville Distribution	91
4.15	References	93
<b>5</b>	<b>Multiresolution Analysis</b>	<b>94</b>
5.1	Multiresolution Spaces	95
5.2	Orthogonal, Biorthogonal, and Semiorthogonal Decomposition	97
5.3	Two-Scale Relations	101
5.4	Decomposition Relation	102
5.5	Spline Functions and Properties	103
5.5.1	Properties of Splines	107
5.6	Mapping a Function into MRA Space	108
5.6.1	Linear Splines ( $m = 2$ )	109
5.6.2	Cubic Splines ( $m = 4$ )	109
5.7	Exercises	110
5.8	Computer Programs	112
5.8.1	B-splines	112
5.9	References	113
<b>6</b>	<b>Construction of Wavelets</b>	<b>114</b>
6.1	Necessary Ingredients for Wavelet Construction	115
6.1.1	Relationship between the Two-Scale Sequences	115
6.1.2	Relationship between Reconstruction and Decomposition Sequences	117
6.2	Construction of Semiorthogonal Spline Wavelets	119
6.2.1	Expression for $\{g_0[k]\}$	119
6.2.2	Remarks	121

6.3	Construction of Orthonormal Wavelets	123
6.4	Orthonormal Scaling Functions	124
6.4.1	Shannon Scaling Function	124
6.4.2	Meyer Scaling Function	126
6.4.3	Battle-Lemarié Scaling Function	129
6.4.4	Daubechies Scaling Function	130
6.5	Construction of Biorthogonal Wavelets	136
6.6	Graphical Display of Wavelet	138
6.6.1	Iteration Method	138
6.6.2	Spectral Method	139
6.6.3	Eigenvalue Method	140
6.7	Exercises	141
6.8	Computer Programs	144
6.8.1	Daubechies Wavelet	144
6.8.2	Iteration Method	145
6.9	References	146
<b>7</b>	<b>DWT and Filter Bank Algorithms</b>	<b>147</b>
7.1	Decimation and Interpolation	147
7.1.1	Decimation	148
7.1.2	Interpolation	151
7.1.3	Convolution Followed by Decimation	154
7.1.4	Interpolation Followed by Convolution	154
7.2	Signal Representation in the Approximation Subspace	155
7.3	Wavelet Decomposition Algorithm	157
7.4	Reconstruction Algorithm	159
7.5	Change of Bases	161
7.6	Signal Reconstruction in Semiorthogonal Subspaces	164
7.6.1	Change of Basis for Spline Functions	164
7.6.2	Change of Basis for Spline Wavelets	168
7.7	Examples	170
7.8	Two-Channel Perfect Reconstruction Filter Bank	172
7.8.1	Spectral-Domain Analysis of a Two-Channel PR Filter Bank	176
7.8.2	Time-Domain Analysis	184
7.9	Polyphase Representation for Filter Banks	189
7.9.1	Signal Representation in Polyphase Domain	189
7.9.2	Filter Bank in the Polyphase Domain	189
7.10	Comments on DWT and PR Filter Banks	191
7.11	Exercises	192
7.12	Computer Programs	193
7.12.1	Decomposition and Reconstruction Algorithm	193
7.13	References	195

<b>8</b>	<b>Special Topics in Wavelets and Algorithms</b>	<b>197</b>
8.1	Fast Integral Wavelet Transform	198
8.1.1	Finer Time Resolution	198
8.1.2	Finer Scale Resolution	201
8.1.3	Function Mapping into the Interoctave Approximation Subspaces	204
8.1.4	Examples	207
8.2	Ridgelet Transform	220
8.3	Curvelet Transform	222
8.4	Complex Wavelets	224
8.4.1	Linear Phase Biorthogonal Approach	227
8.4.2	Quarter-Shift Approach	228
8.4.3	Common Factor Approach	228
8.5	Lifting Wavelet Transform	229
8.5.1	Linear Spline Wavelet	233
8.5.2	Construction of Scaling Function and Wavelet from Lifting Scheme	234
8.5.3	Linear Interpolative Subdivision	234
8.6	References	237
<b>9</b>	<b>Digital Signal Processing Applications</b>	<b>239</b>
9.1	Wavelet Packet	240
9.2	Wavelet-Packet Algorithms	243
9.3	Thresholding	246
9.3.1	Hard Thresholding	246
9.3.2	Soft Thresholding	246
9.3.3	Percentage Thresholding	247
9.3.4	Implementation	247
9.4	Interference Suppression	248
9.4.1	Best Basis Selection	249
9.5	Faulty Bearing Signature Identification	252
9.5.1	Pattern Recognition of Acoustic Signals	252
9.5.2	Wavelets, Wavelet Packets, and FFT Features	254
9.6	Two-Dimensional Wavelets and Wavelet Packets	256
9.6.1	Two-Dimensional Wavelets	256
9.6.2	Two-Dimensional Wavelet Packets	258
9.6.3	Two-Dimensional Wavelet Algorithm	259
9.6.4	Wavelet Packet Algorithm	262
9.7	Edge Detection	264
9.7.1	Sobel Edge Detector	265
9.7.2	Laplacian of Gaussian Edge Detector	265
9.7.3	Canny Edge Detector	266
9.7.4	Wavelet Edge Detector	266

9.8	Image Compression	267
9.8.1	Basics of Data Compression	267
9.8.2	Wavelet Tree Coder	271
9.8.3	EZW Code	272
9.8.4	EZW Example	272
9.8.5	Spatial Oriented Tree (SOT)	275
9.8.6	Generalized Self-Similarity Tree (GST)	277
9.9	Microcalcification Cluster Detection	277
9.9.1	CAD Algorithm Structure	278
9.9.2	Partitioning of Image and Nonlinear Contrast Enhancement	278
9.9.3	Wavelet Decomposition of the Subimages	278
9.9.4	Wavelet Coefficient Domain Processing	279
9.9.5	Histogram Thresholding and Dark Pixel Removal	281
9.9.6	Parametric ART2 Clustering	282
9.9.7	Results	282
9.10	Multicarrier Communication Systems (MCCS)	284
9.10.1	OFDM Multicarrier Communication Systems	284
9.10.2	Wavelet Packet-Based MCCS	285
9.11	Three-Dimensional Medical Image Visualization	287
9.11.1	Three-Dimensional Wavelets and Algorithms	288
9.11.2	Rendering Techniques	290
9.11.3	Region of Interest	291
9.11.4	Summary	291
9.12	Geophysical Applications	293
9.12.1	Boundary Value Problems and Inversion	294
9.12.2	Well Log Analysis	295
9.12.3	Reservoir Data Analysis	296
9.12.4	Downhole Pressure Gauge Data Analysis	298
9.13	Computer Programs	299
9.13.1	Two-Dimensional Wavelet Algorithms	299
9.13.2	Wavelet Packet Algorithms	303
9.14	References	305

## **10 Wavelets in Boundary Value Problems 308**

10.1	Integral Equations	309
10.2	Method of Moments	313
10.3	Wavelet Techniques	314
10.3.1	Use of Fast Wavelet Algorithm	314
10.3.2	Direct Application of Wavelets	315
10.3.3	Wavelets in Spectral Domain	317
10.3.4	Wavelet Packets	322
10.4	Wavelets on the Bounded Interval	322
10.5	Sparsity and Error Considerations	324

10.6	Numerical Examples	327
10.7	Semiorthogonal versus Orthogonal Wavelets	334
10.8	Differential Equations	335
	10.8.1 Multigrid Method	336
	10.8.2 Multiresolution Time Domain (MRTD) Method	337
	10.8.3 Haar-MRTD Derivation	338
	10.8.4 Subcell Modeling in MRTD	341
	10.8.5 Examples	343
10.9	Expressions for Splines and Wavelets	346
10.10	References	348
	<b>Index</b>	<b>353</b>