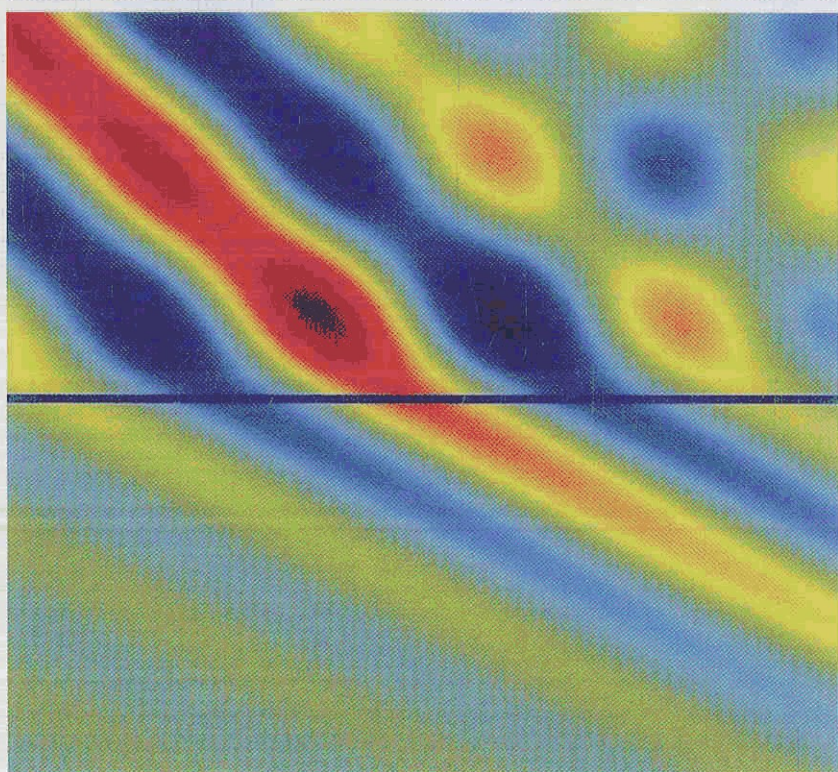


# THEORY AND COMPUTATION OF ELECTROMAGNETIC FIELDS



JIAN-MING JIN

 WILEY

 **IEEE**  
IEEE PRESS

# CONTENTS

<i>PREFACE</i>	xi
<i>ACKNOWLEDGMENTS</i>	xv

## **PART I ELECTROMAGNETIC FIELD THEORY**

### **CHAPTER 1 BASIC ELECTROMAGNETIC THEORY** 3

1.1 Review of Vector Analysis	3
1.1.1 Vector Operations and Integral Theorems	3
1.1.2 Symbolic Vector Method	6
1.1.3 Helmholtz Decomposition Theorem	8
1.1.4 Green's Theorems	9
1.2 Maxwell's Equations in Terms of Total Charges and Currents	9
1.2.1 Maxwell's Equations in Integral Form	11
1.2.2 Maxwell's Equations in Differential Form	14
1.2.3 Current Continuity Equation	14
1.2.4 The Lorentz Force Law	15
1.3 Constitutive Relations	15
1.3.1 Electric Polarization	15
1.3.2 Magnetization	17
1.3.3 Electric Conduction	19
1.3.4 Classification of Media	19
1.4 Maxwell's Equations in Terms of Free Charges and Currents	22
1.5 Boundary Conditions	24
1.6 Energy, Power, and Poynting's Theorem	26
1.7 Time-Harmonic Fields	29
1.7.1 Time-Harmonic Fields	29
1.7.2 Fourier Transforms	30
1.7.3 Complex Power	32
1.7.4 Complex Permittivity and Permeability	36
Notes	37
References	37
Problems	37

### **CHAPTER 2 ELECTROMAGNETIC RADIATION IN FREE SPACE** 43

2.1 Scalar and Vector Potentials	43
----------------------------------	----

2.1.1 Static Fields	44
2.1.2 Time-Harmonic Fields and the Lorenz Gauge Condition	45
2.2 Solution of Vector Potentials in Free Space	47
2.2.1 Delta Function and Green's Function	48
2.2.2 Green's Function in Free Space	49
2.2.3 Field-Source Relations in Free Space	50
2.2.4 Why Use Auxiliary Potential Functions	51
2.2.5 Free-Space Dyadic Green's Functions	52
2.3 Electromagnetic Radiation in Free Space	55
2.3.1 Infinitesimal Electric Dipole	55
2.3.2 Finite Electric Dipole	57
2.3.3 Far-Field Approximation and the Sommerfeld Radiation Condition	59
2.3.4 Circular Current Loop and Magnetic Dipole	61
2.4 Radiation by Surface Currents and Phased Arrays	63
2.4.1 Radiation by a Surface Current	63
2.4.2 Radiation by a Phased Array	65
Notes	69
References	69
Problems	69

### **CHAPTER 3 ELECTROMAGNETIC THEOREMS AND PRINCIPLES** 73

3.1 Uniqueness Theorem	73
3.2 Image Theory	75
3.2.1 Basic Image Theory	76
3.2.2 Half-Space Field-Source Relations	80
3.3 Reciprocity Theorems	82
3.3.1 General Reciprocity Theorem	82
3.3.2 Lorentz Reciprocity Theorem	83
3.3.3 Rayleigh-Carson Reciprocity Theorem	83
3.4 Equivalence Principles	85
3.4.1 Surface Equivalence Principle	85

3.4.2	Application to Scattering by a Conducting Object	87
3.4.3	Application to Scattering by a Dielectric Object	90
3.4.4	Volume Equivalence Principle	92
3.5	Duality Principle	94
3.6	Aperture Radiation and Scattering	95
3.6.1	Equivalent Problems	96
3.6.2	Babinet's Principle	99
3.6.3	Complementary Antennas	101
	References	102
	Problems	103
<b>CHAPTER 4 TRANSMISSION LINES AND PLANE WAVES</b>		
		<b>107</b>
4.1	Transmission Line Theory	107
4.1.1	Governing Differential Equations and General Solutions	107
4.1.2	Reflection and Transmission	110
4.1.3	Green's Function and Eigenfunction Expansion	111
4.2	Wave Equations and General Solutions	115
4.2.1	Wave Equations and Solution by Separation of Variables	116
4.2.2	Characteristics of a Plane Wave	117
4.2.3	Wave Velocities and Attenuation	119
4.2.4	Linear, Circular, and Elliptical Polarizations	121
4.2.5	Wave Propagation in Metamaterials	124
4.3	Plane Waves Generated by A Current Sheet	125
4.4	Reflection and Transmission	127
4.4.1	Reflection and Transmission at Normal Incidence	127
4.4.2	Reflection and Transmission at Oblique Incidence	129
4.4.3	Total Transmission and Total Reflection	132
4.4.4	Transmission into a Left-Handed Medium	135
4.4.5	Plane Waves versus Transmission Lines	137
4.5	Plane Waves in Anisotropic and Bi-Isotropic Media	137
4.5.1	Plane Waves in Uniaxial Media	137
4.5.2	Plane Waves in Gyrotropic Media	142
4.5.3	Plane Waves in Chiral Media	145
	References	147
	Problems	147

**CHAPTER 5 FIELDS AND WAVES IN RECTANGULAR COORDINATES** **152**

---

5.1	Uniform Waveguides	152
5.1.1	General Analysis	152
5.1.2	General Characteristics	156
5.1.3	Uniform Rectangular Waveguide	160
5.1.4	Losses in Waveguides and Attenuation Constant	167
5.2	Uniform Cavities	170
5.2.1	General Theory	171
5.2.2	Rectangular Cavity	173
5.2.3	Material and Geometry Perturbations	175
5.3	Partially Filled Waveguides and Dielectric Slab Waveguides	178
5.3.1	General Theory	178
5.3.2	Partially Filled Rectangular Waveguide	180
5.3.3	Dielectric Slab Waveguide on a Ground Plane	183
5.4	Field Excitation in Waveguides	187
5.4.1	Excitation by Planar Surface Currents	187
5.4.2	Excitation by General Volumetric Currents	189
5.5	Fields in Planar Layered Media	191
5.5.1	Spectral Green's Function and Sommerfeld Identity	191
5.5.2	Vertical Electric Dipole above a Layered Medium	192
5.5.3	Horizontal Electric Dipole above a Layered Medium	194
5.5.4	Dipoles on a Grounded Dielectric Slab	196

Note 197

References 198

Problems 198

**CHAPTER 6 FIELDS AND WAVES IN CYLINDRICAL COORDINATES** **200**

---

6.1	Solution of Wave Equation	200
6.1.1	Solution by Separation of Variables	200
6.1.2	Cylindrical Wave Functions	203
6.2	Circular and Coaxial Waveguides and Cavities	204
6.2.1	Circular Waveguide	205
6.2.2	Coaxial Waveguide	208
6.2.3	Cylindrical Cavity	212
6.3	Circular Dielectric Waveguide	213
6.3.1	Analysis of Hybrid Modes	214

6.3.2 Characteristics of Hybrid Modes 217

6.4 Wave Transformation and Scattering Analysis 222

6.4.1 Wave Transformation 223

6.4.2 Scattering by a Circular Conducting Cylinder 224

6.4.3 Scattering by a Circular Dielectric Cylinder 227

6.4.4 Scattering by a Circular Multilayer Dielectric Cylinder 230

6.5 Radiation by Infinitely Long Currents 234

6.5.1 Line Current Radiation in Free Space 234

6.5.2 Radiation by a Cylindrical Surface Current 236

6.5.3 Radiation in the Presence of a Circular Conducting Cylinder 238

6.5.4 Radiation in the Presence of a Conducting Wedge 241

6.5.5 Radiation by a Finite Current 243

References 245

Problems 245

**CHAPTER 7 FIELDS AND WAVES IN SPHERICAL COORDINATES** 248

7.1 Solution of Wave Equation 248

7.1.1 Solution by Separation of Variables 248

7.1.2 Spherical Wave Functions 251

7.1.3 TE<sub>n</sub> and TM<sub>n</sub> Modes 253

7.2 Spherical Cavity 255

7.3 Biconical Antenna 259

7.3.1 Infinitely Long Model 259

7.3.2 Finite Biconical Antenna 262

7.4 Wave Transformation and Scattering Analysis 264

7.4.1 Wave Transformation 264

7.4.2 Expansion of a Plane Wave 266

7.4.3 Scattering by a Conducting Sphere 269

7.4.4 Scattering by a Dielectric Sphere 274

7.4.5 Scattering by a Multilayer Dielectric Sphere 276

7.5 Addition Theorem and Radiation Analysis 281

7.5.1 Addition Theorem for Spherical Wave Functions 281

7.5.2 Radiation of a Spherical Surface Current 283

7.5.3 Radiation in the Presence of a Sphere 286

7.5.4 Radiation in the Presence of a Conducting Cone 288

References 291

Problems 292

**PART II ELECTROMAGNETIC FIELD COMPUTATION**

**CHAPTER 8 THE FINITE DIFFERENCE METHOD** 297

8.1 Finite Differencing Formulas 298

8.2 One-Dimensional Analysis 300

8.2.1 Solution of the Diffusion Equation 300

8.2.2 Solution of the Wave Equation 302

8.2.3 Stability Analysis 302

8.2.4 Numerical Dispersion Analysis 305

8.3 Two-Dimensional Analysis 306

8.3.1 Analysis in the Time Domain 306

8.3.2 Analysis in the Frequency Domain 308

8.4 Yee's FDTD Scheme 309

8.4.1 Two-Dimensional Analysis 309

8.4.2 Three-Dimensional Analysis 311

8.5 Absorbing Boundary Conditions 314

8.5.1 One-Dimensional ABC 314

8.5.2 Two-Dimensional ABCs 316

8.5.3 Perfectly Matched Layers 318

8.6 Modeling of Dispersive Media 329

8.6.1 Recursive Convolution Approach 329

8.6.2 Auxiliary Differential Equation Approach 331

8.7 Wave Excitation and Far-Field Calculation 333

8.7.1 Modeling of Wave Excitation 334

8.7.2 Near-to-Far Field Transformation 337

8.8 Summary 338

References 339

Problems 340

**CHAPTER 9 THE FINITE ELEMENT METHOD** 342

9.1 Introduction to the Finite Element Method 343

9.1.1 The General Principle 343

9.1.2 One-Dimensional Example 344

9.2 Finite Element Analysis of Scalar Fields 347

9.2.1 The Boundary-Value Problem 347

9.2.2 Finite Element Formulation 348

9.2.3 Application Examples 354

9.3	Finite Element Analysis of Vector Fields	359
9.3.1	The Boundary-Value Problem	359
9.3.2	Finite Element Formulation	360
9.3.3	Application Examples	364
9.4	Finite Element Analysis in the Time Domain	372
9.4.1	The Boundary-Value Problem	373
9.4.2	Finite Element Formulation	374
9.4.3	Application Examples	378
9.5	Absorbing Boundary Conditions	379
9.5.1	Two-Dimensional ABCs	380
9.5.2	Three-Dimensional ABCs	383
9.5.3	Perfectly Matched Layers	386
9.6	Some Numerical Aspects	390
9.6.1	Mesh Generation	391
9.6.2	Matrix Solvers	391
9.6.3	Higher-Order Elements	392
9.6.4	Curvilinear Elements	393
9.6.5	Adaptive Finite Element Analysis	393
9.7	Summary	393
	References	394
	Problems	395
<b>CHAPTER 10 THE METHOD OF MOMENTS 399</b>		
10.1	Introduction to the Method of Moments	399
10.2	Two-Dimensional Analysis	404
10.2.1	Formulation of Integral Equations	404
10.2.2	Scattering by a Conducting Cylinder	407
10.2.3	Scattering by a Conducting Strip	411
10.2.4	Scattering by a Homogeneous Dielectric Cylinder	415
10.3	Three-Dimensional Analysis	416
10.3.1	Formulation of Integral Equations	416
10.3.2	Scattering and Radiation by a Conducting Wire	421
10.3.3	Scattering by a Conducting Body	425
10.3.4	Scattering by a Homogeneous Dielectric Body	430
10.3.5	Scattering by an Inhomogeneous Dielectric Body	433
10.4	Analysis of Periodic Structures	436
10.4.1	Scattering by a Planar Periodic Conducting Patch Array	436
10.4.2	Scattering by a Discrete Body-of-Revolution Object	441
10.5	Analysis of Microstrip Antennas and Circuits	443
10.5.1	Formulation of Integral Equations	443
10.5.2	The Moment-Method Solution	446
10.5.3	Evaluation of Green's Functions	446
10.5.4	Far-Field Calculation and Application Examples	451
10.6	The Moment Method in the Time Domain	452
10.6.1	Time-Domain Integral Equations	453
10.6.2	Marching-On-in-Time Solution	454
10.7	Summary	458
	References	458
	Problems	460
<b>CHAPTER 11 FAST ALGORITHMS AND HYBRID TECHNIQUES 463</b>		
11.1	Introduction to Fast Algorithms	463
11.2	Conjugate Gradient-FFT Method	465
11.2.1	Scattering by a Conducting Strip or Wire	466
11.2.2	Scattering by a Conducting Plate	466
11.2.3	Scattering by a Dielectric Object	472
11.3	Adaptive Integral Method	478
11.3.1	Planar Structures	478
11.3.2	Three-Dimensional Objects	482
11.4	Fast Multipole Method	487
11.4.1	Two-Dimensional Analysis	487
11.4.2	Three-Dimensional Analysis	492
11.4.3	Multilevel Fast Multipole Algorithm	495
11.5	Adaptive Cross-Approximation Algorithm	500
11.5.1	Low-Rank Matrix	500
11.5.2	Adaptive Cross-Approximation	502
11.5.3	Application to the Moment-Method Solution	504
11.6	Introduction to Hybrid Techniques	508
11.7	Hybrid Finite Difference-Finite Element Method	509
11.7.1	Relation between FETD and FDTD	510
11.7.2	Hybridization of FETD and FDTD	512
11.7.3	Application Example	514
11.8	Hybrid Finite Element-Boundary Integral Method	515
11.8.1	Traditional Formulation	517

11.8.2	Symmetric Formulation	520
11.8.3	Numerical Examples	523
11.9	Summary	526
	Notes	527
	References	527
	Problems	531
<b>CHAPTER 12</b>	<b><i>CONCLUDING REMARKS ON COMPUTATIONAL ELECTROMAGNETICS</i></b>	<b>533</b>
<hr/>		
12.1	Overview of Computational Electromagnetics	533
12.1.1	Frequency- versus Time-Domain Analysis	533
12.1.2	High-Frequency Asymptotic Techniques	534
12.1.3	First-Principle Numerical Methods	536

12.1.4	Time-Domain Simulation Methods	538
12.1.5	Hybrid Techniques	540
12.2	Applications of Computational Electromagnetics	540
12.3	Challenges in Computational Electromagnetics	551
	References	552

---

***APPENDIX*** **559**

Vector Identities	559
Integral Theorems	559
Coordinate Transformation	560

***INDEX*** **561**