

**Steven C. Chapra**

**Applied Numerical Methods**  
with **MATLAB<sup>®</sup>**

*for Engineers and Scientists*

**Third Edition**

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