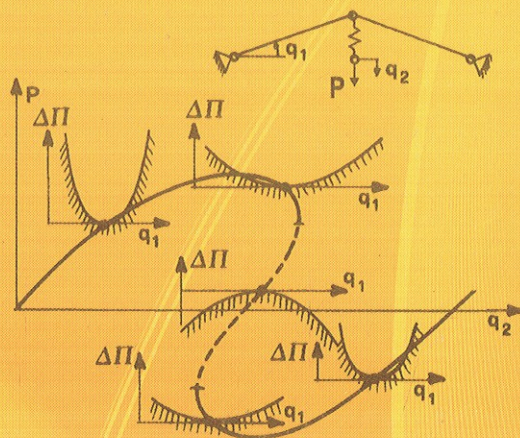


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# STABILITY OF STRUCTURES

Elastic, Inelastic, Fracture  
and Damage Theories





# Contents

<b>Preface</b>	<b>vii</b>
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<b>Introduction</b>	<b>xxi</b>
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## I ELASTIC THEORIES

<b>1 Buckling of Elastic Columns by Equilibrium Analysis</b>	<b>3</b>
1.1 Theory of Bending	4
1.2 Euler Load, Adjacent Equilibrium, and Bifurcation	5
1.3 Differential Equations of Beam-Columns	9
1.4 Critical Loads of Perfect Columns with Various End Restraints	12
1.5 Imperfect Columns and the Southwell Plot	19
Lateral Disturbing Load; Initial Curvature or Load Eccentricity; Behavior near the Critical Load; Southwell Plot	
1.6 Code Specifications for Beam-Columns	27
1.7 Effect of Shear and Sandwich Beams	30
Pin-Ended Columns; Generalization; Sandwich Beams and Panels	
1.8 Pressurized Pipes and Prestressed Columns	35
Pressurized Pipes; Prestressed Columns	
1.9 Large Deflections	38
Solution of Rotations by Elliptic Integrals (Elastica); Deflections and Shortening; Discussion of Results	
1.10 Spatial Buckling of Beams under Torque and Axial Force	46
<b>2 Buckling of Elastic Frames by Equilibrium Analysis</b>	<b>53</b>
2.1 Stiffness and Flexibility Matrices of Beam-Columns	54
Stiffness Matrix for End Rotations; Stiffness Matrix for End Rotations and Relative Lateral Displacement	
2.2 Critical Loads of Frames and Continuous Beams	61
Simple Structures; Difficulties with the Flexibility Method; General Approach for Computer Analysis	
2.3 Buckling as a Matrix Eigenvalue Problem and Use of Finite Elements	74
2.4 Large Regular Frames	78
2.5 Postcritical Reserve in Redundant Trusses	86
Example of a Statically Indeterminate Truss; Generalization and Limit Analysis Method; Order of Approximation	

2.6	Postcritical Behavior of Frames	89
	L-Frame of Koiter and Roorda; Second-Order Solution of the L-Frame; Imperfection Sensitivity; Generalizations and Implications; Order of Approximation; Postcritical Reserve Due to Redundancy; Finite Element Computational Procedure	
2.7	Built-Up Columns and Regular Frames as Columns with Shear	102
2.8	High Arches	108
	Curvature Change; Approximate Theory for Perfect Arches with a Fixed Compression Line; Various Types of Arches and the Effect of Imperfections; General Linearized Theory for Uniformly Compressed Circular Arches	
2.9	Long-Wave Buckling of Regular Frames	118
	System of Difference Equations; Solution for Tall Building Frames; Numerical Results and Their Discussion	
2.10	Continuum Approximation for Large Regular Frames	129
	Transition from Difference to Differential Equations; Continuum Approximation Based on Potential Energy; Micropolar Continuum and Couple Stresses; Stresses and Boundary Conditions; Numerical Results and Discussion	
<b>3</b>	<b>Dynamic Analysis of Stability</b>	<b>144</b>
3.1	Vibration of Columns or Frames and Divergence	144
	Columns; Types of Motion and Dependence of Natural Frequency on Load; Effect of Damping; Frames and Other Generalizations	
3.2	Nonconservative Loads and Flutter	151
	Massless Column under Follower Load; Effect of Distributed Mass; Elastically Supported Rigid Plate under Aerodynamic Forces; Conservative and Nonconservative Forces; Equations Governing Flutter or Suspension Bridges	
3.3	Pulsating Loads and Parametric Resonance	162
	Axial Pulsating Load on a Column; Undamped Vibration; Damped Vibration; Simple Energy Analysis of Parametric Resonance	
3.4	Other Types of Dynamic Loads	170
3.5	Definition of Stability	174
3.6	Theorems of Lagrange–Dirichlet and of Liapunov	178
3.7	Stability Criteria for Dynamic Systems	184
3.8	Stability of Continuous Elastic Systems	187
3.9	Nonlinear Oscillations and Chaos	189
<b>4</b>	<b>Energy Methods</b>	<b>199</b>
4.1	Positive-Definite Matrices, Eigenvalues, and Eigenvectors	199
4.2	Potential Energy for Discrete Elastic Systems	207
	Structure-Load System; Second Variation of Potential Energy; Critical State; An Example; Effect of Higher-Order Derivatives of $\Pi$ ; Difficulties with Complementary Energy; Overturning Instability of a Block: Discontinuous $\Pi'$	
4.3	Bifurcation Buckling at Small Deflections	216
	Calculation of Potential Energy of Beam-Columns; Equilibrium	

	and Stability; Role of Axial Strain and Shortening due to Deflections; Calculation of $\Pi$ from the Work of Initial and Incremental Stresses; Example with Two Degrees of Freedom; Some Fundamental Aspects of Potential Energy	
4.4	<b>Snapthrough and Flat Arches</b> Von Mises Truss; Flat Arches; Effect of Imperfections; Other Examples of Snapthrough	227
4.5	<b>Large-Deflection Postcritical Behavior and Types of Bifurcation</b> Symmetric Stable Bifurcation: Example; Symmetric Unstable Bifurcation: Example; Asymmetric Bifurcation: Example; L-Shaped Rigid-Bar Frame; Rigid-Bar Arch; Nonlinear Springs and Polynomial Approximation of Potential Energy; Two Degrees of Freedom: Example; Limit Points of Equilibrium Paths; Bifurcation Criterion in Terms of the Tangential Stiffness Matrix; Classification of Elementary Instabilities of Elastic Structures	238
4.6	<b>Koiter's Theory, Imperfection Sensitivity, and Interaction of Modes</b> General Validity of Koiter's $\frac{1}{2}$ -Power and $\frac{2}{3}$ -Power Laws; Interaction of Buckling Modes	261
4.7	<b>Catastrophe Theory and Breakdown of Symmetry</b>	270
4.8	<b>Snapdown at Displacement-Controlled Loading</b> Structures with Two Degrees of Freedom; Softening Specimen in a Testing Machine; Generalization of Snapdown Analysis; Equilibrium Paths with Bifurcations, Snapthrough, and Snapdown	278
4.9	<b>Incremental Work Criterion at Equilibrium Displacements</b> Stability Criterion; Possibility of Generalization to Inelastic Systems	294
<b>5</b>	<b>Energy Analysis of Continuous Structures and Approximate Methods</b>	<b>305</b>
5.1	<b>Indirect Variational Method and Euler Equation</b> Review of the Calculus of Variations; Application to Structures Possessing a Potential Energy; Review of Positive-Definite and Self-Adjoint Operators	306
5.2	<b>Beam on Elastic Foundation</b> Potential Energy and Differential Equations; Solution for Different Boundary Conditions; Fiber on Elastic Foundation	314
5.3	<b>Rayleigh Quotient</b> Upper-Bound Property of Rayleigh Quotient; Application to Beam-Columns; Relation to Differential Equation; Proof of Upper-Bound Property and Convergence; Extension to Free Vibration	323
5.4	<b>Timoshenko Quotient and Relations between Various Bounds</b> Derivation; Examples; Relation to Differential Equation and Proof of Upper-Bound Property; Relation to Rayleigh Quotient and Inequalities; Inapplicability to Dynamics; The Question of Lower Bounds	331
5.5	<b>Bound Approximation for Columns, Frames, and High Arches</b> Columns; Frames; Elastically Supported Beams; High Arches	339

5.6	Rayleigh–Ritz Variational Method	348
5.7	Galerkin Variational Method	356
5.8	Method of Successive Approximations and Lower Bounds	358
	Formulation of the Method; Example; Lower Bound	
5.9	Nonlinear Problems; Large Deflections of Columns	361
<b>6</b>	<b>Thin-Walled Beams</b>	<b>370</b>
6.1	Potential Energy and Differential Equations	371
	Deformation of the Cross Section; Potential Energy; Differential Equations and Boundary Conditions	
6.2	Axial-Torsional Buckling of Columns	381
6.3	Lateral Buckling of Beams and Arches	384
	Axial-Torsional Buckling due to Eccentric Axial Force; Lateral Buckling due to Bending Moment; Approximate Solution for Variable $M_z^0$ ; Bimoment; Lateral Buckling of Arches	
6.4	Beams of Arbitrary Open Cross Section	392
	General Theory of Warping Torsion; Stresses and Bimoment in General Theory; Potential Energy and Differential Equations; Monosymmetric Cross Section	
6.5	Large Deflections	401
6.6	Box Girders	408
	Deformation Modes and Postcritical Energy; Examples; Finite Element Solution; Interaction with Local Buckling	
<b>7</b>	<b>Plates and Shells</b>	<b>419</b>
7.1	Classical Plate Theory	420
7.2	Differential Equation and Strain Energy	422
	Strains; Potential Energy; Differential Equations of Equilibrium; Boundary Conditions; Direct Derivation of Transverse Resultant of In-Plane Forces; Discussion and Summary	
7.3	Buckling of Rectangular Plates	431
	Buckling of Simply Supported Plates; Rectangular Plate with Arbitrary Boundary Conditions; Buckling of Plate Subjected to Shear; Nonuniform In-Plane Forces; Solutions by Other Variational Methods	
7.4	Large Deflections and Postcritical Reserve of Plates	440
	Von Kármán–Föppl Differential Equations; Solution by Minimization of Potential Energy; Large Deflections and Ultimate Strength; Measurement of Critical Loads	
7.5	Axisymmetric Buckling of Cylindrical Shells	449
7.6	Shallow or Quasi-Shallow Shells	453
	Basic Relations for Cylindrical Shells; Donnell's Equation; Axially Compressed Cylindrical Shell; Effect of Lateral Pressure on Cylindrical Shells; Cylindrical Shell Subjected to Torsion; Variational Derivation from Potential Energy; Cylindrical Shell Panels; General Quasi-Shallow Shells	
7.7	Nonlinear Analysis of Shell Buckling and Imperfections	466
	Reduction Factors for Classical Critical Loads; Physical Source of Postcritical Load Drop and High Imperfection Sensitivity; Koiter's	

	Laws of Imperfection Sensitivity; Buckling Modes and Their Interaction; Summary	
7.8	<b>Sandwich Plates and Shells</b>	474
	Basic Relations for a Sandwich Plate Element; Rectangular Sandwich Plate and Other Problems	

## II INELASTIC, DAMAGE, AND FRACTURE THEORIES

<b>8</b>	<b>Elastoplastic Buckling</b>	<b>485</b>
8.1	Perfect Columns or Structures and Shanley's Bifurcation	486
	Reduced Modulus Load; Tangent Modulus Load; Column Strength Curve; Postbifurcation Load-Deflection Diagram; Bifurcation in Plastic Structures with Multiaxial Stress; Conclusion	
8.2	Imperfect Columns and Structures	506
	Shanley's Rigid-Bar Column: Exact Solution; Arbitrary Imperfect Columns: Approximate Solution; Effect of Cross-Section Nonsymmetry	
8.3	Effect of Residual Stresses	513
	Calculation of the Effect of Residual Stresses; Examples	
8.4	Metal Columns and Structures: Design and Code Specifications	517
	Centrically Loaded Columns; Load and Resistance Factor Design and Probabilistic Aspects; Beam-Columns; Plates, Shells, and Other Structures; Design Examples	
8.5	Concrete Columns and Structures: Design and Code Specifications	532
	Interaction Diagram (Failure Envelope); Deflections and Interaction Diagram; Numerical Algorithm for Calculating Deflections and Interaction Diagram; Column Response for Unsmooth Stress-Strain Diagrams; Design Recommendations and the ACI Code; CEB Design Recommendations; Comparisons of Codes and Shortcomings; Prestressed Concrete Columns; Shells and Other Structures; Stress-Strain Relations for Strain Softening; Design Examples	
8.6	Perfectly Plastic Large-Deflection Buckling, Impact, and Blast	556
	Load-Deflection Curve or Perfectly Plastic Columns; Buckling of Perfectly Plastic Frames; Plastic Redistribution and Reserve Capacity of Structures; Dynamic Impact; Perfectly Plastic Buckling of Thick Plates; Transverse Impact or Blast on Plates or Columns with In-Plane or Axial Loads	
8.7	Geometric Tensile Instability, Localization, and Necking	569
	Role of Transverse Contraction and Finite Strain; Strain Localization; Necking	
<b>9</b>	<b>Creep Buckling</b>	<b>584</b>
9.1	Viscoelastic Stress-Strain Relations	585
	Compliance Function and Integral-Type Creep Law; Differential-Type Creep Law and Rheologic Models; Elastic-Viscoelastic Analogy	

9.2	Viscoelastic Buckling	590
	Deflection History and Long-Time Critical Load; The Concept of Stability for Viscoelastic Structures; Extensions and Ramifications	
9.3	Viscoplastic Buckling	597
	Rigid-Bar Model Column; Critical Time and Stability Concept; Real Columns	
9.4	Buckling of Aging Viscoelastic Structures	603
	Aging Maxwell Solid (Dischinger-Type Methods); Deflections According to Aging Maxwell Model; Deflection According to More Realistic Rheologic Models; Deflection According to Effective Modulus; Deflection According to Age-Adjusted Effective Modulus; Deflection According to Integral-Type Stress-Strain Relation; Appendix I—Compliance Function and Relaxation Function of Concrete; Appendix II—Proof of Age-Adjusted Effective Modulus Method	
9.5	Effect of Creep Deflection on Concrete Column Strength	615
9.6	Nonlinear Creep and Long-Time Strength of Concrete Structures	619
9.7	Creep Buckling at Finite Deflections	623
	Example of Imperfection-Sensitive Rigid-Bar Column; Broader Implications and Ramifications; Variable Load	
10	<b>Stability of Inelastic Structures, Bifurcation and Thermodynamic Basis</b>	<b>633</b>
10.1	Thermodynamic Criteria of Stable State	633
	First and Second Laws of Thermodynamics; Tangentially Equivalent Elastic Structures; Total Energy $U$ and Helmholtz Free Energy $F$ ; Second Variation of $\mathcal{F}$ or $\mathcal{U}$ ; Path Dependence and Incremental Potentials; Second-Order Work of Stresses and Geometric Stiffness; Criterion of Stable State for the Case of Dead Loads; Extensions to Variable Loads; Stability at Critical State; Gibbs Free Energy and Enthalpy; Stability Criteria Based on Complementary Work; Structures with a Single Load or a Single Controlled Displacement; Summary	
10.2	Thermodynamic Criteria of Stable Path	650
	Path Stability for Basic Types of Control; Mixed Controls of Loads and Displacements; The Case of Equal $(\Delta S)_{in}$ for Two Branches; Second-Order Work of Stresses along the Path; Structures with a Single Load or a Single Controlled Displacement; Stable States on Postbifurcation Branches; Further Comments and Conclusion	
10.3	Application to Elastoplastic Columns and Broader Implications	658
	Loading-Unloading Combinations and Equilibrium Paths; Second-Order Work; Stable Equilibrium States of Elastoplastic Column; Stable Equilibrium Path of an Elastoplastic Column; Breakdown of Symmetry; Hypothesis Implied in Present Thermodynamic Approach; Summary	
10.4	Critical States of Stability and Bifurcation	670
	Critical State for Structures with a Symmetric Stiffness Matrix; Critical States for Structures with a Nonsymmetric Stiffness	

	Matrix; Example of a Nonsymmetric Stiffness Matrix; Symmetric and Asymmetric Bifurcations at the Critical State; Uniqueness; Bifurcation for Inelastic Structures and Hill's Linear Comparison Solid; Distribution of Bifurcation Points and Postbifurcation Branches; Numerical Finite Element Analysis; Summary	
10.5	Stability at Infinitesimal Loading Cycles Internal Entropy Changes for Cycles in Shanley's Column; Stability; Structures with a Single Cyclic Load or Displacement; Incremental Collapse	680
10.6	Drucker's and Il'yushin's Postulates for Stable Materials Drucker's Postulate; Il'yushin's Postulate; Nonuniformly Strained Bodies; Normality Rule for Plasticity	685
10.7	Stability of Frictional Materials and Structures Frictional Block Preloaded by a Spring; Generalization to Frictional Continuum; Stability Condition of Frictional Materials; Plastic Strain Increment for Frictional Materials; Inverse Material Friction; Frictional Phenomena in Other Constitutive Theories	693
<b>11</b>	<b>Three-Dimensional Continuum Instabilities and Effects of Finite Strain Tensor</b>	<b>706</b>
11.1	Finite Strain Notations and Basic Requirements; Lagrangian (Green's) Finite Strain Tensor; Biot's Finite Strain Tensor; Second-Order Approximations of Other Finite Strain Tensors; Further Measures of Finite Strain; The Special Case of Thin Bodies; Decomposition of Strain into Elastic and Inelastic Parts	707
11.2	Stresses, Work, and Equilibrium at Finite Strain Virtual Work Relations and Equilibrium; True (Cauchy) Stress; Stress Referred to Initial Configuration and Working on Displacement Gradient; Stress Referred to Initial Configuration and Working on Finite Strain	715
11.3	Incremental Equilibrium and Objective Stress Rates Incremental Equilibrium Conditions; Increments of Cauchy (True) Stresses; Objective Stress Increments Conjugate to Strain Increments; Objective Stress Rates	720
11.4	Tangential Moduli at Large Initial Stress	726
11.5	Stable States and Paths for Multidimensional Continuous Bodies	732
11.6	Column or Plate with Shear: Finite-Strain Effect Differential Equations; Theories of Engesser and Haringx; Correlation to Built-Up Columns; Summary	734
11.7	Surface Buckling and Internal Buckling of Anisotropic Solids Basic Relations for Incompressible Orthotropic Solids; Surface Buckling of an Orthotropic Half-Space; Internal Buckling and Other Instabilities; General Solution	741
11.8	Consistent Geometric Stiffness Matrix of Finite Elements	748
11.9	Buckling of Curved Fibers in Composites Macroscopic Elastic Stress-Strain Relations; Decrease of Elastic Moduli Due to Fiber Undulation; Generalization to Three Dimensions; Stresses Due to Fiber Undulation	750



<b>12 Fracture as a Stability Problem</b>	<b>760</b>
12.1 Linear Elastic Fracture Mechanics	760
Stress Singularity and Fracture Energy; Energy Release Rate; Determination of $\mathcal{G}$ and $G_f$ from Compliance Changes; Some Simple Elastic Solutions; Approximation by Stress Relief Zone; Examples Solvable by Bending Theory; Herrmann's Method to Obtain Approximate $K_I$ by Beam Theory	
12.2 Nonlinear Fracture Mechanics and Size Effect	772
Inelastic Zone and Equivalent Elastic Crack; Fracture Models with a Nonlinear Zone; Size Effect	
12.3 Crack Stability Criterion and <i>R</i> -Curve	782
<i>R</i> -Curve and Fracture Equilibrium Condition; Fracture Stability Criterion and Critical State; Determination of Geometry-Dependent <i>R</i> -Curve from Size Effect Law; Crack Propagation Direction; Kinking of Cracks and Three-Dimensional Instability of Front Edge	
12.4 Snapback Instability of a Crack and Ligament Tearing	788
General Procedure for Load-Displacement Relation at Growing Crack; Snapback Instability at Crack Coalescence in Two Dimensions; Snapback Instability at Tearing of Circular Ligament; General Condition for Snapback at Ligament Tearing; Alternative Calculation of Displacement from Compliance Variation	
12.5 Stable States and Stable Paths of Interacting Cracks	798
Conditions of Equilibrium and Stability in Terms of Crack Length; Stability of Parallel Cooling or Shrinkage Cracks; Stable Path and Bifurcation at Advancing Cooling Front; Three-Dimensional Pattern of Cooling or Shrinkage Cracks; Stability of Parallel Cracks in Reinforced Concrete; Stability Analysis in Terms of Displacements	
12.6 Crack Spacing	816
Spacing of Parallel Initial Drying or Cooling Cracks; Energy Balance; Size of Initial Hexagonal Cracking Cells from Energy Balance; Snapthrough Formation of Cracks According to LEFM; Crack Spacing in Loaded Reinforced Concrete Beams; Snapthrough Crack Formation in a Drying Tube	
<b>13 Damage and Localization Instabilities</b>	<b>829</b>
13.1 Wave in Strain-Softening Materials	830
Exact Solution of Strain-Softening Bar; Stability Aspects and Unrealistic Properties of Solution of a Bar; Bar with Rehardening or Residual Yield Strength; Cylindrically or Spherically Converging Waves; General Three-Dimensional Condition for Waves to Exist; Summary	
13.2 Series-Coupling Model for Localization Due to Softening	845
Stable States; Surface of Second-Order Work; Application to Uniaxially Stressed Bars or Specimens; Effects of Size and Support Stiffness; Specimen Ductility as a Function of Length and Loading Frame Stiffness; Inadmissibility of Arbitrarily Small Size of Localization Region; Bifurcation and Stable Path; Alternative: Imperfection Approach to Bifurcation; Identification	

	of Softening Stress-Strain Relations from Tests; Relation of Strain Softening to Fracture Energy; Summary	
13.3	Localization of Softening Damage into Planar Bands Stability Condition for the Softening Band Within a Layer or Infinite Solid; Discussion of Various Cases; Numerical Examples; Generalization for Geometrically Nonlinear Effects; Bifurcation and Stable Path; Localization into Shear Bands Due to Nonassociatedness in Frictional Materials; Sand Liquefaction as a Localization Instability; Summary	857
13.4	Localization of Softening Damage into Ellipsoidal Regions Eshelby's Theorem; Stability of Uniform Strain against Ellipsoidal Localization; Numerical Examples of Stability Limits and Discussion; Bifurcation and Stable Path of Ellipsoidal Localization; Simpler Derivation of Bifurcation Condition; Summary	869
13.5	Localization of Softening Damage into Spherical or Circular Regions Localization Instability for Spherical Geometry; Localization Instability for Circular or Cylindrical Geometry; Numerical Examples; Bifurcation and Stable Path; Summary	880
13.6	Localization in Beams and Softening Hinges Stability Limit and Snapback; Rotation Capacity or Ductility of Hinges in Concrete Beams; Length of the Softening Region; Bifurcation Due to Interaction of Softening Hinges; Imperfection Approach; Bifurcation and Localization in Redundant Structures; Bifurcation at Simultaneous Formation of Several Softening Hinges; Softening Frames and Trusses; Softening in Metallic Structures; Summary	887
13.7	Friction: Static and Dynamic Paradox in Sudden Friction Drop; Bifurcation, Stable Path, and Localization of Frictional Slip; Frictional Supports in Columns; Structures with Stiffness Matrix Asymmetry	902
13.8	Bifurcations Due to Interaction of Softening Damage Zones Interaction of Damage (Cracking) Fronts and Stable Paths; Convergence of Iterations Depends on Stability of State, not Path; Multiple Interacting Crack Band Fronts; Interaction of Multiple Shear Bands; Example: Buckling in Direct Tensile Test	907
13.9	Size Effect, Mesh Sensitivity, and Energy Criterion for Crack Bands Localization as a Cause of Size Effect; Inobjectivity or Spurious Mesh Sensitivity; Energy Criterion for Crack Band and Stability	914
13.10	Nonlocal Continuum and Its Stability Crack Band Model; Nonlocal Continuum Concept; Periodic Instabilities Due to Nonlocal Concept; Nonlocal Continuum with Local Strain; One-Dimensional Localization Instability; Measurement of Characteristic Length of Nonlocal Continuum; Example: Stability of a Tunnel; Gradient Approximation to Nonlocal Continuum; Summary	921
13.11	Constitutive Equations for Strain Softening	937

<b>Glossary of Symbols</b>	<b>953</b>
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<b>Author Index</b>	<b>963</b>
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<b>Subject Index</b>	<b>973</b>
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