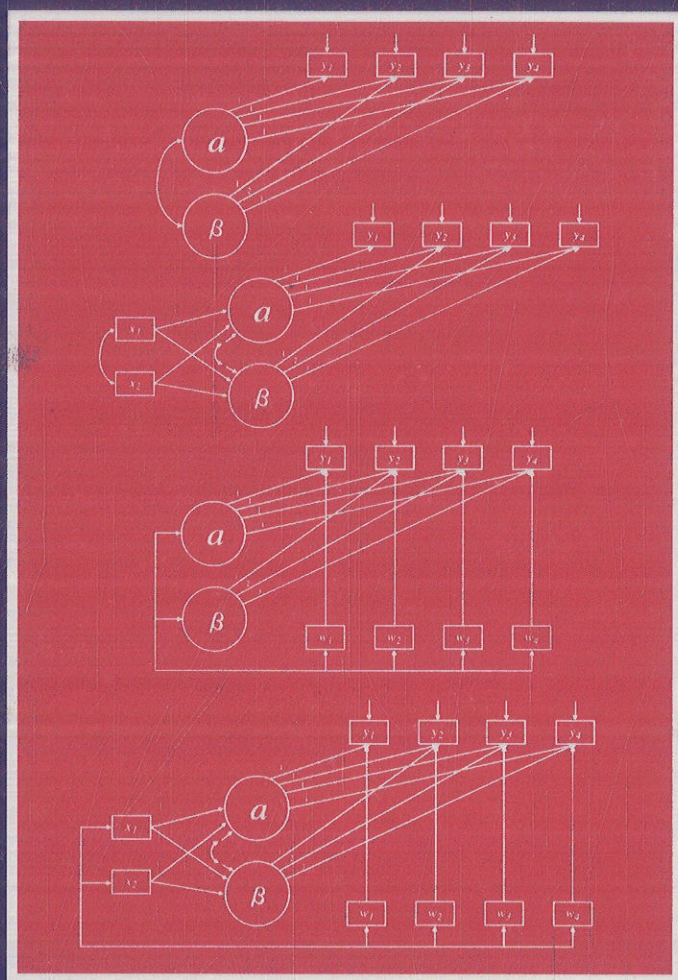


# Latent Curve Models

## A Structural Equation Perspective



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