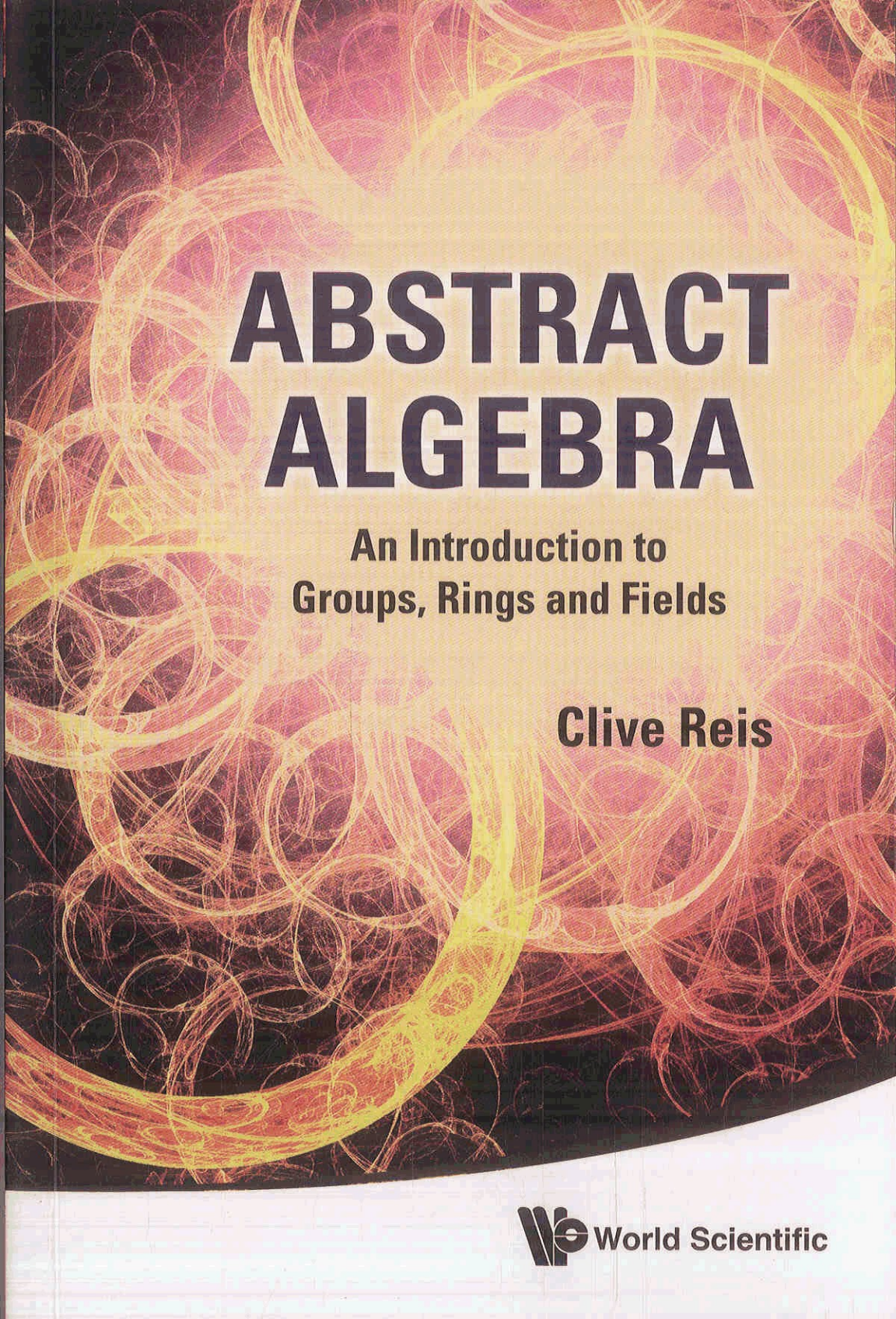




ABSTRACT ALGEBRA

**An Introduction to
Groups, Rings and Fields**

Clive Reis

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