

Classical Complex Analysis

A Geometric Approach — Vol.2

I-Hsiung Lin



World Scientific

Contents

Volume 2

Chapter 5	Fundamental Theory: Sequences, Series, and Infinite Products	1
5.1	Power Series	3
5.1.1	Algorithm of power series	3
5.1.2	Basic properties of an analytic function defined by a power series in an open disk	16
5.1.3	Boundary behavior of a power series on its circle of convergence	24
5.2	Analytic Continuation	49
5.2.1	Analytic continuation along a curve	52
5.2.2	Homotopy and monodromy theorem	60
5.3	Local Uniform Convergence of a Sequence or a Series of Analytic Functions	63
5.3.1	Analyticity of the limit function: Weierstrass's theorem	64
5.3.2	Zeros of the limit function: Hurwitz's theorem	82
*5.3.3	Some Sufficient Criteria for Local Uniform Convergence	82
*5.3.4	An application: The fixed points of an analytic function and its iterate functions	82
5.4	Meromorphic Functions: Mittag-Leffler's Partial Fractions Theorem	121
5.4.1	Mittag-Leffler's partial fractions expansion for meromorphic functions	122
5.4.2	Cauchy's residue method	133
5.5	Entire Functions: Weierstrass's Factorization Theorem and Hadamard's Order Theorem	147
5.5.1	Infinite products (of complex numbers and functions)	150
5.5.2	Weierstrass's factorization theorem	161
5.5.3	Hadamard's order theorem	172

5.6	The Gamma Function $\Gamma(z)$	183
5.6.1	Definition and representations	184
5.6.2	Basic and characteristic properties	197
5.6.3	The asymptotic function of $\Gamma(z)$; Stirling's formula	216
5.7	The Riemann zeta Function $\zeta(z)$	232
5.8	Normal Families of Analytic (Meromorphic) Functions	244
5.8.1	Criteria for normality	245
5.8.2	Examples	252
5.8.3	The elliptic modular function: Montel's normality criterion and Picard's theorems	267
*5.8.4	Remarks on Schottky's Theorems and Schwarz-Ahlfors' Lemma: Other proofs of Montel's criterion and Picard's theorems	284
*5.8.5	An application: Some results in complex dynamical system	300
Chapter 6	Conformal Mapping and Dirichlet's Problems	307
6.1	The Riemann Mapping Theorem	308
6.1.1	Proof	310
6.1.2	The boundary correspondence	318
6.2	Conformal Mapping of Polygons: The Schwarz-Christoffel Formulas	328
6.2.1	The Schwarz-Christoffel formulas for polygons	329
6.2.2	Examples	345
6.2.3	The Schwarz-Christoffel formula for the generalized polygon	367
6.3	Harmonic Function and the Dirichlet Problem for a Disk	391
6.3.1	Some further properties of harmonic functions: Green's function	392
6.3.2	Poisson's formula and integral: The Dirichlet problem for a disk; Harnack's principle	406
6.4	Subharmonic Functions	427
6.5	Perron's Method: Dirichlet's Problem for a Class of General Domains	433
6.6	Canonical Mappings and Canonical Domains of Finitely Connected Domains	443
6.6.1	Harmonic measures	448
6.6.2	Canonical domains: The annuli with concentric circular slits	456

6.6.3	Canonical domains: The parallel slit domains	464
*6.6.4	Other canonical domains	471
Chapter 7	Riemann Surfaces (Abstract)	484
7.1	Riemann Surface: Definition and Examples	486
7.2	Analytic Mappings and Meromorphic Functions on Riemann Surfaces	498
7.3	Harmonic Functions and the Maximum Principle on Riemann Surfaces	531
7.3.1	Harmonic measure	536
7.3.2	Green's function	543
7.3.3	A classification of Riemann surfaces	557
7.4	The Fundamental Group	565
7.5	Covering Spaces (or Surfaces) and Covering Transformations	570
7.5.1	Definitions and examples	570
7.5.2	Basic properties of covering spaces (or surfaces): The lifting of mappings and the monodromy theorem	578
7.5.3	Characteristics and classifications of covering surfaces: The existence theorem	590
7.5.4	Covering transformations	600
7.5.5	The universal covering surface of a surface	610
7.6	The Uniformization Theorem of Riemann Surfaces	616
7.6.1	Simply connected Riemann surfaces	620
7.6.2	Arbitrary Riemann surfaces	626
Appendix B.	Parabolic, Elliptic, and Hyperbolic Geometries	631
Appendix C.	Quasiconformal Mappings	663
References	674
Index of Notations	678
Index	685