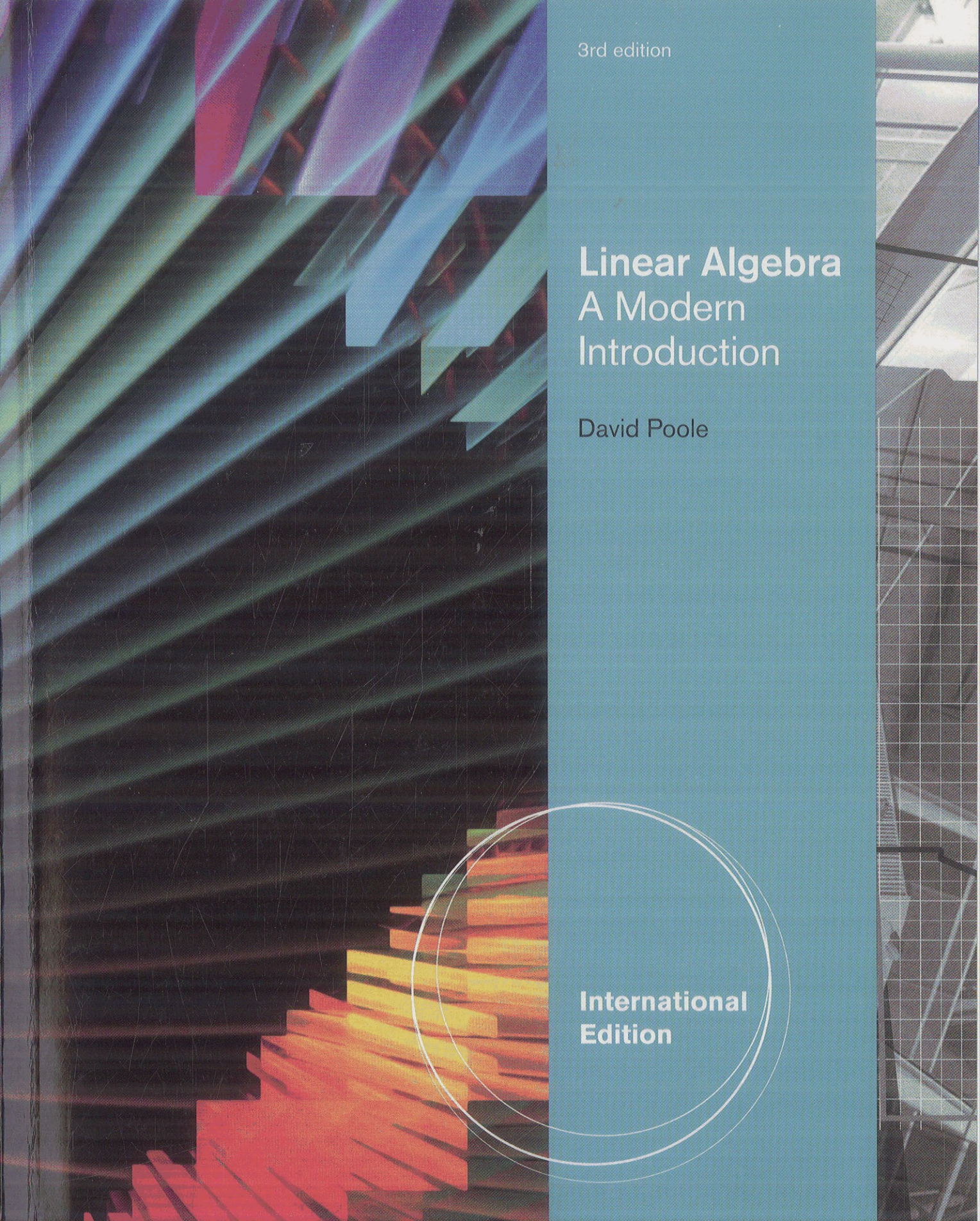

3rd edition

Linear Algebra
A Modern Introduction

David Poole

International Edition

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