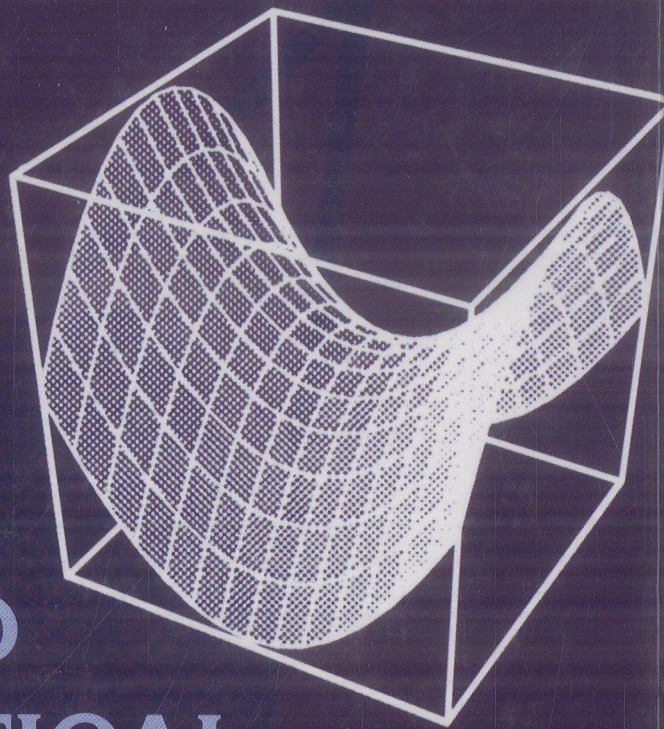




ADVANCED MATHEMATICAL METHODS IN SCIENCE AND ENGINEERING

Second Edition

S. I. Hayek



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