

Third Edition

DISCRETE-TIME SIGNAL PROCESSING



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with a companion website by Mark A. Yoder and Wayne T. Padgett

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