Contents

Preface **vii**

1 Predicate Logic **1**

1.1 Introduction 1

Biography: Bertrand Russell 6

1.2 Propositional logic 7

1.3 Quantifiers 14

1.4 First-order languages and theories 23

Biography: Euclid 27

1.5 Examples of first-order theories 32

1.6 Normal forms and complexity 38

1.7 Other logics 47

2 Axiomatic Set Theory **59**

2.1 Introduction 59

2.2 "Naive" set theory 61

Biography: Georg Ferdinand Cantor 62

2.3 Zermelo–Fraenkel set theory 68

Biography: John von Neumann 72

2.4 Ordinals 77

2.5 Cardinals and the cumulative hierarchy 84

3 Recursion Theory and Computability **95**
3.1 Introduction 95
Biography: Emil Post 97
3.2 Primitive recursive functions 98
3.3 Turing machines and recursive functions 108
Biography: Alan Turing 109
3.4 Undecidability and recursive enumerability 118
3.5 Complexity theory 125

4 Gödel's Incompleteness Theorems **135**
4.1 Introduction 135
4.2 The arithmetization of formal theories 136
Biography: Kurt Gödel 137
4.3 A potpourri of incompleteness theorems 146
4.4 Strengths and limitations of PA 156
Biography: Frank Ramsey 159

5 Model Theory **165**
5.1 Introduction 165
5.2 Basic concepts of model theory 167
5.3 The main theorems of model theory 172
5.4 Preservation theorems 181
5.5 Saturation and complete theories 191
Biography: Julia Robinson 198
5.6 Quantifier elimination 205
5.7 Additional topics in model theory 214

6 Contemporary Set Theory **225**
6.1 Introduction 225
6.2 The relative consistency of AC and GCH 228
6.3 Forcing and the independence results 234
6.4 Modern set theory and large cardinals 242
6.5 Descriptive set theory 254

Biography: Nikolai Luzin 261
6.6 The axiom of determinacy 267

7 Nonstandard Analysis **279**
7.1 Introduction 279
Biography: Archimedes 282
Biography: Leonhard Euler 287
7.2 Nonarchimedean fields 289
7.3 Standard and nonstandard models 293
7.4 Nonstandard methods in mathematics 304

8 Constructive Mathematics **317**
8.1 Introduction 317
8.2 Brouwer's intuitionism 325
Biography: L. E. J. Brouwer 326
8.3 Bishop's constructive analysis 330

A A Deductive System for First-order Logic **347**

B Relations and Orderings **349**

C Cardinal Arithmetic **355**

D Groups, Rings, and Fields **361**

Bibliography **375**

Symbols and Notation **381**

Index **387**