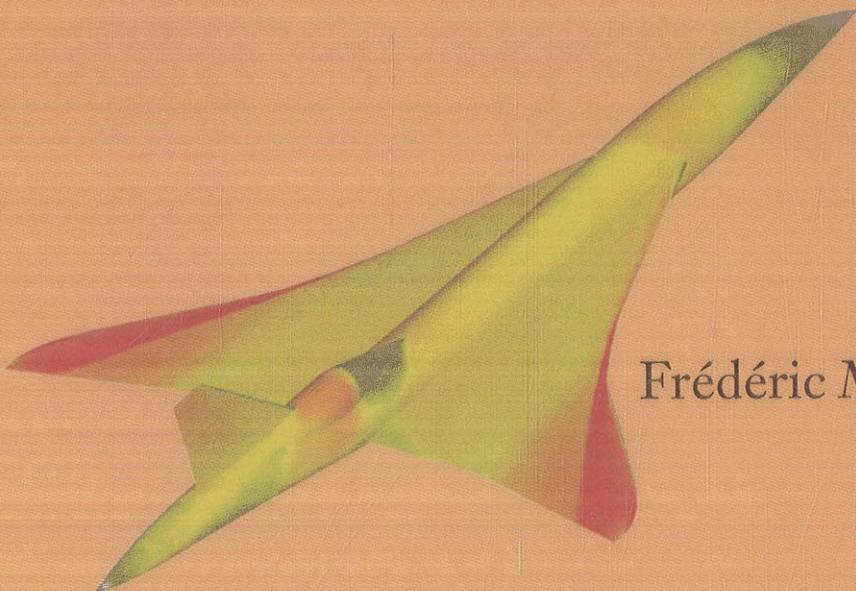


Chapman & Hall/CRC  
Numerical Analysis and Scientific Computing

# Computational Fluid Dynamics



Edited by  
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CRC Press  
Taylor & Francis Group

A CHAPMAN & HALL BOOK

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