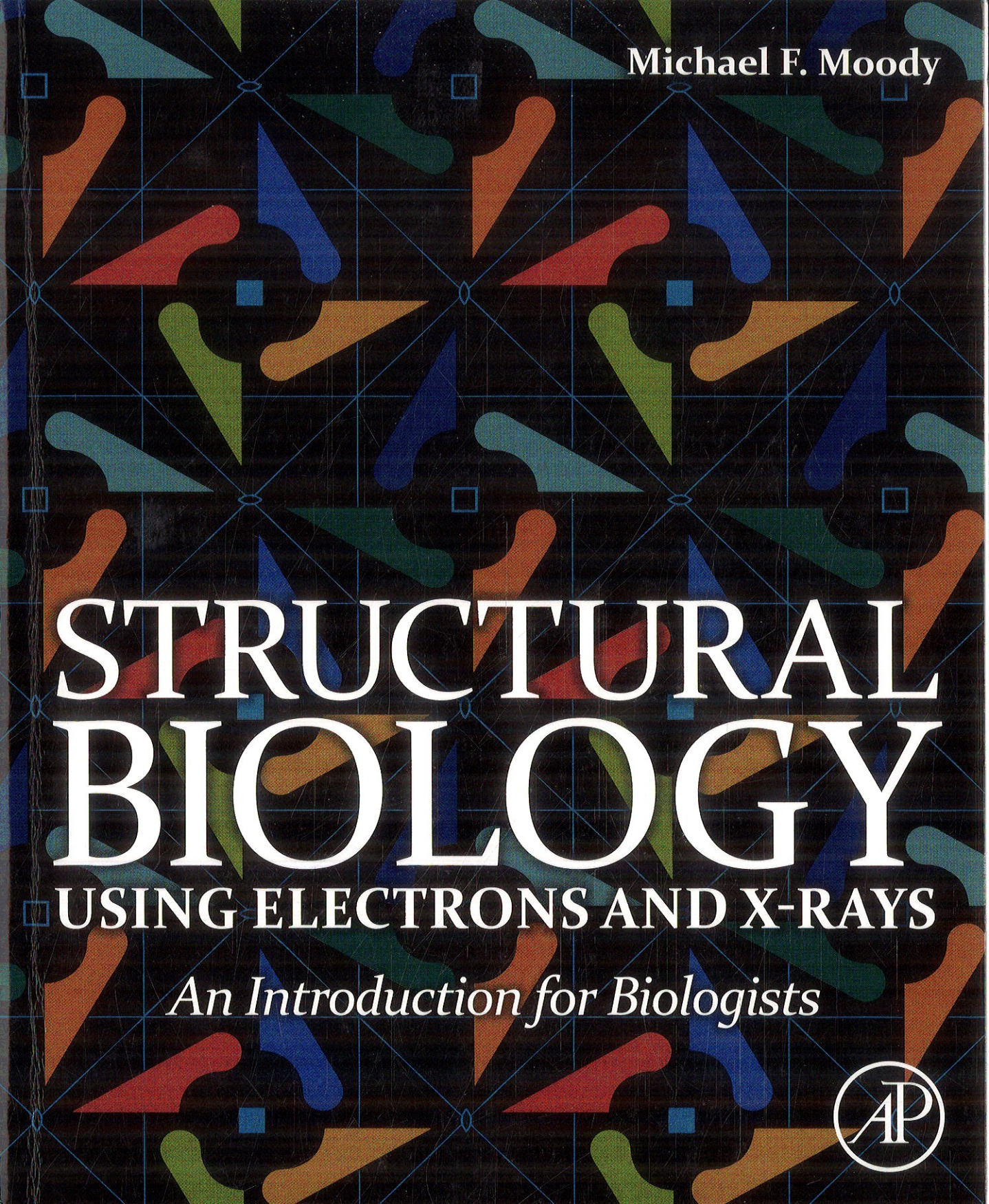


The background of the book cover features a complex, repeating geometric pattern. It consists of a dark grid of thin blue lines forming squares and diamonds. Overlaid on this grid are various colorful shapes: triangles, quadrilaterals, and irregular polygons in shades of blue, green, orange, and red. Some of these shapes are solid, while others are outlined or semi-transparent, creating a layered, crystalline effect. The overall aesthetic is scientific and modern.

Michael F. Moody

STRUCTURAL BIOLOGY

USING ELECTRONS AND X-RAYS

An Introduction for Biologists



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