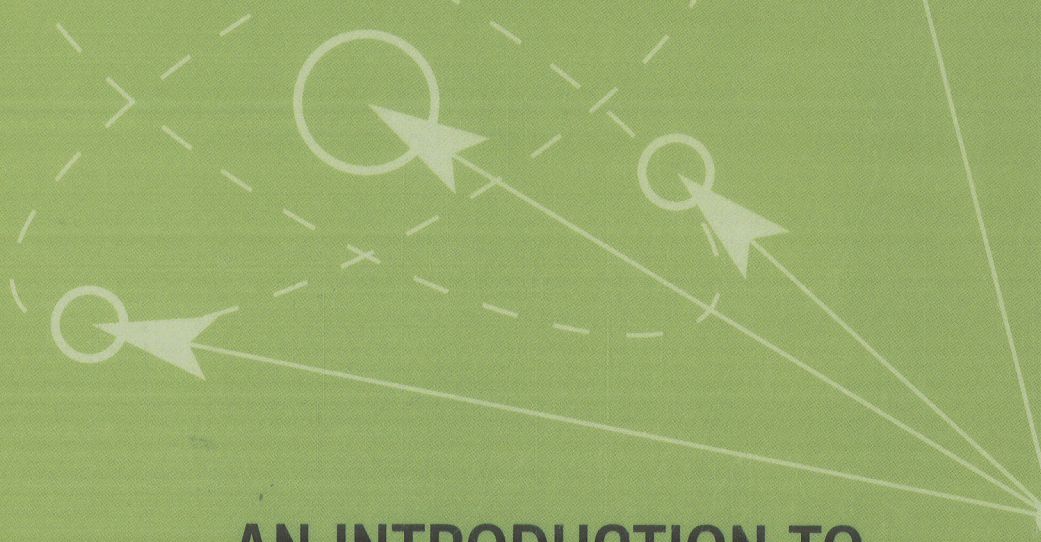
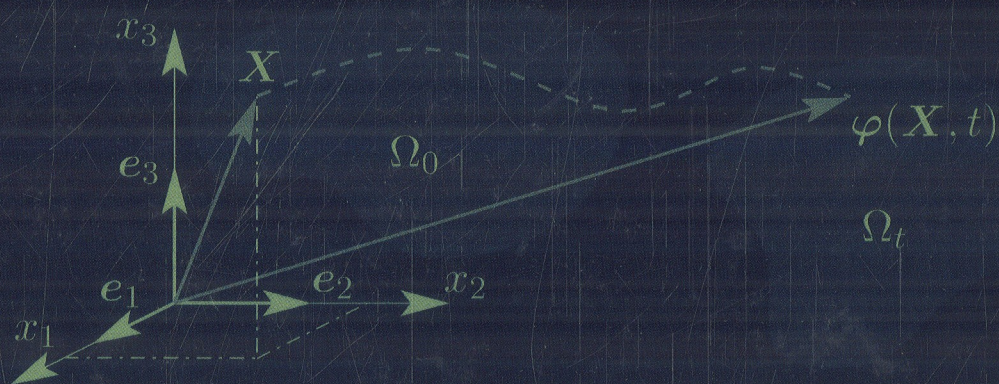


WILEY SERIES IN COMPUTATIONAL MECHANICS



AN INTRODUCTION TO Mathematical Modeling

A COURSE IN MECHANICS



J. TINSLEY ODEN

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