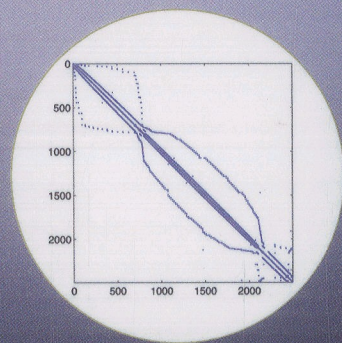
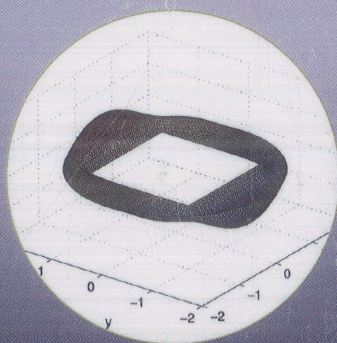
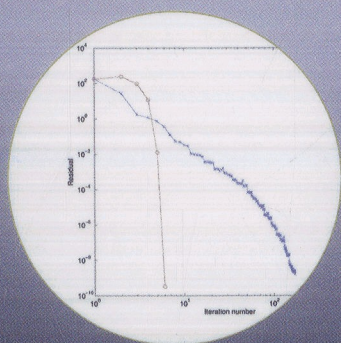


Pure and Applied Mathematics:
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NUMERICAL ANALYSIS *of* PARTIAL DIFFERENTIAL EQUATIONS



S.H. LUI

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