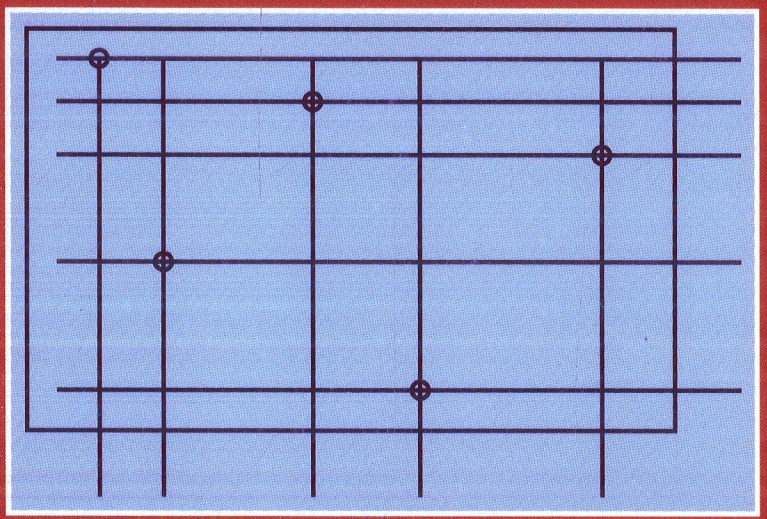
CAMBRIDGE TEXTS IN APPLIED MATHEMATICS

Iterative Methods in Combinatorial Optimization



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CAMBRIDGE

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