

## CONTENTS

Preface	xiii
1. Sets	1
1.1 Preliminaries . . . . .	1
1.2 Algebra of Sets . . . . .	4
1.3 Venn Diagrams . . . . .	9
1.4 Power Set . . . . .	10
1.5 Countable Sets . . . . .	11
1.6 Some Special Maps (Functions) . . . . .	17
1.6.1 The characteristic function . . . . .	21
1.7 Partitions of Sets . . . . .	22
1.8 The Minset and Maxset Normal Forms . . . . .	24
1.9 Multisets . . . . .	31
2. Propositional Calculus and Logic	41
2.1 Propositions . . . . .	41
2.2 Compositions of Propositions . . . . .	43
2.3 Truth Tables and Applications . . . . .	45
2.4 Some Further Applications of Logic . . . . .	55
2.5 Functionally Complete Set of Connectives . . . . .	63
2.6 The Connectives NAND and NOR . . . . .	64
3. More on Sets	69
3.1 The Principle of Inclusion and Exclusion . . . . .	69
3.2 The Pigeonhole Principle . . . . .	81
3.2.1 Some typical applications of the pigeonhole principle . . . . .	82

3.3	Binary Relations . . . . .	89
3.3.1	Relations . . . . .	89
3.3.2	Equivalence relations . . . . .	89
3.3.3	Union, intersection and inverse of relations . . .	91
3.3.4	Composition of relations . . . . .	93
3.3.5	The matrix of a relation . . . . .	95
3.3.6	Closure operations on relations . . . . .	98
4.	Some Counting Techniques . . . . .	107
4.1	The Principle of Mathematical Induction . . . . .	107
4.2	Strong Induction . . . . .	114
4.3	Arithmetic, Geometric and Arithmetic-Geometric Series . . . . .	131
4.4	Permutations and Combinations . . . . .	144
4.4.1	Rules of product and sum . . . . .	144
4.4.2	Permutations . . . . .	147
4.4.3	The arrangements of objects that are not all distinct . . . . .	149
4.4.4	Combinations . . . . .	152
4.4.5	Generation of permutations and combinations . . . . .	156
5.	Recurrence Relations . . . . .	167
5.1	Partial Fractions . . . . .	167
5.1.1	Rational functions . . . . .	167
5.1.2	Partial fractions . . . . .	168
5.1.3	Procedure for resolving into partial fractions . . . . .	170
5.1.4	Some solved examples . . . . .	173
5.2	Recurrence Relations: Preliminaries . . . . .	180
5.2.1	Homogeneous solutions . . . . .	183
5.2.2	Particular solutions . . . . .	187
5.2.3	Solution by the method of generating functions . . . . .	193
5.2.4	Some typical examples . . . . .	197
5.2.5	Recurrence relations reducible to linear recurrence relations . . . . .	205

6.	Partially Ordered Sets	213
6.1	Preliminaries . . . . .	213
6.2	Hasse Diagrams . . . . .	216
6.3	Chains and Antichains in Posets . . . . .	221
7.	Graphs	241
7.1	Preliminaries and Graph Terminology . . . . .	241
7.1.1	Some typical examples . . . . .	261
7.2	Paths and Circuits . . . . .	265
7.3	Shortest Path in Weighted Graphs . . . . .	271
7.4	Eulerian Paths and Circuits . . . . .	281
7.5	Hamiltonian Paths and Circuits . . . . .	298
7.6	Planar Graphs . . . . .	309
7.6.1	Applications . . . . .	313
7.6.2	Some further examples . . . . .	316
7.6.3	Graph colouring . . . . .	319
7.7	Matrix Representations of Graphs . . . . .	325
7.7.1	Adjacency matrix . . . . .	325
8.	Trees	343
8.1	Introduction and Elementary Properties . . . . .	343
8.2	Rooted Trees . . . . .	350
8.3	Tree Searching or Traversing a Tree . . . . .	362
8.4	Applications of Trees . . . . .	376
8.4.1	Prefix codes . . . . .	376
8.4.2	Binary search trees . . . . .	384
8.4.3	On counting trees . . . . .	388
8.4.4	Some further examples . . . . .	395
8.5	Spanning Trees and Cut-Sets . . . . .	400
8.6	Minimal/Minimum/Shortest Spanning Tree . . . . .	416
9.	Groups	443
9.1	Groups: Preliminaries . . . . .	443
9.2	Subgroups . . . . .	449
9.2.1	Lagrange's theorem . . . . .	451
9.3	Quotient Groups . . . . .	455
9.4	Symmetric Groups . . . . .	457

10.	Rings	467
10.1	Rings . . . . .	467
10.2	Polynomial Rings . . . . .	470
10.3	Quotient Rings and Homomorphisms . . . . .	474
11.	Fields and Vector Spaces	481
11.1	Fields . . . . .	481
11.1.1	Field extensions and minimal polynomial . . . . .	484
11.1.2	Characteristic of a field . . . . .	485
11.1.3	Splitting field . . . . .	485
11.2	Vector Spaces . . . . .	491
11.2.1	Basis of a vector space . . . . .	494
11.2.2	Subspaces and quotient spaces . . . . .	498
11.2.3	Linear transformations . . . . .	504
12.	Lattices and Boolean Algebra	509
12.1	Lattices . . . . .	509
12.2	Lattices as Algebraic Systems . . . . .	515
12.3	Sublattices and Homomorphisms . . . . .	521
12.4	Distributive and Modular Lattices . . . . .	525
12.5	Complemented Lattices . . . . .	541
12.6	Boolean Algebras . . . . .	545
12.7	Boolean Polynomials and Boolean Functions . . . . .	554
12.8	Switching (or Logical) Circuits . . . . .	567
13.	Matrices, Systems of Linear Equations and Eigen Values	577
13.1	Linear System of Equations . . . . .	577
13.1.1	Rank of a matrix . . . . .	577
13.1.2	Linear system of equations . . . . .	578
13.2	Elementary Row Operations, Gaussian Elimination . . . . .	581
13.2.1	Elementary row operations . . . . .	581
13.2.2	Gaussian elimination in matrix form . . . . .	583
13.2.3	Gaussian elimination method . . . . .	585
13.2.4	Direct methods for the solution of linear system of equations . . . . .	591

*Contents*

xi

13.2.5	Method of factorization . . . . .	594
13.2.6	Some additional examples . . . . .	598
13.3	Eigen Values . . . . .	600
13.3.1	Eigen values and eigen vectors . . . . .	603
	Bibliography	613
	Index	615