

Contents

<i>Preface</i>	vii
1. Background Material	1
1.1 Introduction	1
1.2 Wiener Process and Wiener Measure	1
1.3 Stochastic Differential Equations in R^d	5
1.4 Stochastic Differential Equations in H	14
1.4.1 Measure Solutions	20
1.5 Nonlinear Filtering	23
1.5.1 Finite Dimensional Filtering	23
1.5.2 Infinite Dimensional Filtering	26
1.6 Elements of Vector Measures	28
1.7 Some Problems for Exercise	35
2. Regular Functionals of Brownian Motion	37
2.1 Introduction	37
2.2 Functionals of Scalar Brownian Motion	38
2.3 Functionals of Vector Brownian Motion	44
2.4 Functionals of Gaussian Random Field (GRF)	49
2.5 Functionals of Multidimensional Gaussian Random Fields	52
2.6 Functionals of ∞ -Dimensional Brownian Motion	56
2.7 Fr-Br. Motion and Regular Functionals Thereof	59
2.8 Lévy Process and Regular Functionals Thereof	64
2.9 Some Problems for Exercise	68

3.	Generalized Functionals of the First Kind I	71
3.1	Introduction	71
3.2	Mild Generalized Functionals I	71
3.3	Mild Generalized Functionals II	76
3.4	Generalized Functionals of GRF I	77
3.5	Generalized Functionals of GRF II	79
3.6	Generalized Functionals of ∞ -Dim. Brownian Motion . .	81
3.7	Generalized Functionals of Fr.Brownian Motion and Lévy Process	82
3.8	Some Problems for Exercise	84
4.	Functional Analysis on $\{G, \mathcal{G}\}$ and Their Duals	87
4.1	Introduction	87
4.2	Compact and Weakly Compact Sets	87
4.3	Some Optimization Problems	90
4.4	Applications to SDE	92
4.5	Vector Measures	97
4.6	Application to Nonlinear Filtering	101
4.7	Application to Infinite Dimensional Systems	107
4.8	Lévy Optimization Problem	111
4.9	Some Problems for Exercise	114
5.	L_2 -Based Generalized Functionals of White Noise II	117
5.1	Introduction	117
5.2	Characteristic Function of White Noise	118
5.3	Multiple Wiener-Itô Integrals	120
5.4	Generalized Hida-Functionals	121
5.5	Application to Quantum Mechanics	125
5.6	Some Problems for Exercise	128
6.	L_p -Based Generalized Functionals of White Noise III	131
6.1	Introduction	131
6.2	Homogeneous Functionals of Degree n	131
6.3	Nonhomogeneous Functionals	135
6.4	Weighted Generalized Functionals	139
6.5	Some Examples Related to Section 6.4	141
6.6	Generalized Functionals of Random Fields Applied . . .	144
6.7	$\tilde{\mathcal{F}}_q$ -Valued Vector Measures with Application	151

6.8	Some Problems for Exercise	157
7.	$\mathcal{W}^{p,m}$ -Based Generalized Functionals of White Noise IV	159
7.1	Introduction	159
7.2	Homogeneous Functionals	159
7.3	Nonhomogeneous Functionals	163
7.4	Inductive and Projective Limits	168
7.5	Abstract Generalized Functionals	170
7.6	Vector Measures with Values from Wiener-Itô Distributions	177
7.7	Application to Nonlinear Filtering	179
7.8	Application to Stochastic Navier-Stokes Equation	184
7.9	Some Problems for Exercise	208
8.	Some Elements of Malliavin Calculus	211
8.1	Introduction	211
8.2	Abstract Wiener Space	212
8.3	Malliavin Derivative and Integration by Parts	213
8.4	Operator δ the Adjoint of the Operator D	219
8.5	Ornstein-Uhlenbeck Operator L	225
8.6	Sobolev Spaces on Wiener Measure Space $\Omega \equiv (\Omega, \mu)$	230
8.7	Smoothness of Probability Measures	233
8.7.1	Smoothness under Ellipticity Condition	236
8.7.2	Smoothness under Hörmander's Conditions	241
8.7.3	Some Illustrative Examples	243
8.7.4	Some Comments on Degeneracy Condition	246
8.8	Central Limit Theorem for Wiener-Itô Functionals	248
8.9	Malliavin Calculus for Fr-Brownian Motion	256
8.10	Some Problems for Exercise	257
9.	Evolution Equations on Fock Spaces	259
9.1	Introduction	259
9.2	Malliavin Operators on Fock Spaces	260
9.3	Evolution Equations on Abstract Fock Spaces	270
9.4	Evolution Equations Determined by Coercive Operators on Fock Spaces	273
9.5	An Example	277
9.6	Evolution Equations on Wiener-Sobolev Spaces	280

9.7	Some Examples for Exercise	289
	<i>Bibliography</i>	291
	<i>Index</i>	297