

Quantum Field Theory and its Macroscopic Manifestations

Boson Condensation, Ordered Patterns
and Topological Defects



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Contents

<i>Preface</i>	vii
1. The structure of the space of the physical states	1
1.1 Introduction	1
1.2 The space of the states of physical particles	2
1.3 The Weyl–Heisenberg algebra and the Fock space	7
1.4 Irreducible representations of the canonical commutation relations	10
1.5 Unitarily equivalent representations	15
1.6 The Stone–von Neumann theorem	17
1.7 Unitarily inequivalent representations	19
1.8 The deformation of Weyl–Heisenberg algebra	21
1.8.1 Self-similarity, fractals and the Fock–Bargmann representation	26
1.9 The physical particle energy and momentum operator	30
1.10 The physical Fock space and the physical fields	32
Appendix A Strong limit and weak limit	35
Appendix B Glauber coherent states	37
Appendix C Generalized coherent states	41
Appendix D q -WH algebra, coherent states and theta functions	49

2. Inequivalent representations of the canonical commutation relations	53
2.1 Introduction	53
2.2 Heisenberg fields, physical fields and the dynamical map	54
2.3 Examples of inequivalent representations	58
2.4 The Haag theorem and non-perturbative physics	66
2.5 The momentum operator	67
2.6 Time evolution and asymptotic limits	68
2.7 Inequivalent representations in flavor mixing	71
Appendix E Computation of $\langle 0 \psi_i(x) \alpha_n\alpha_m\rangle$	85
Appendix F Computation of $ 0(\theta)\rangle$	87
Appendix G Orthogonality of flavor vacua at different times	89
Appendix H Entanglement in neutrino oscillations	91
3. Spontaneous breakdown of symmetry and the Goldstone theorem	97
3.1 Introduction	97
3.2 Invariance and symmetry	99
3.3 Irreducible representations of the symmetry group	101
3.4 Symmetry and the vacuum manifold	102
3.5 Boson transformation and inequivalent representations	106
3.6 Spontaneous symmetry breaking and functional integrals	108
3.7 The Goldstone theorem	112
3.7.1 $U(1)$ symmetry	112
3.7.2 $SU(2)$ symmetry	115
3.8 Spontaneous symmetry breaking in local gauge theories	119
3.8.1 The $U(1)$ local gauge model	119
3.8.2 The chiral gauge model	124
3.9 Finite volume effects	126
3.10 Space-time dimensionality	128
Appendix I The order parameter space	133
Appendix J The Mermin–Wagner–Coleman theorem	135

4.	Dynamical rearrangement of symmetry and macroscopic manifestations of QFT	137
4.1	Introduction	137
4.2	Dynamical rearrangement of symmetry	139
4.2.1	$SU(2)$ symmetry	139
4.2.2	Global $U(1)$ symmetry	143
4.2.3	Local $U(1)$ symmetry and the emergence of classical Maxwell equations	145
4.3	The boson transformation theorem and the non-homogeneous boson condensation	153
4.3.1	Topological singularities, gapless modes and macroscopic observables	156
4.3.2	Defect formation in the process of symmetry breaking phase transitions	158
4.4	Group contraction and spontaneous symmetry breaking	159
4.4.1	The infrared effect	159
4.4.2	Group contraction, boson condensation and macroscopic quantum systems	165
4.4.3	The collective behavior of quantum components and group contraction	166
4.5	Quantum fluctuations and macroscopic stability	169
4.5.1	Quantum mechanical decoherence and stability of macroscopic quantum systems	174
	Appendix K Group contraction and Virasoro algebra	179
	Appendix L Phase locking in the N atom system	183
5.	Thermal field theory and trajectories in the space of the representations	185
5.1	Introduction	185
5.2	Doubling the degrees of freedom	188
5.2.1	The two-slit experiment	189
5.3	Thermo Field Dynamics: A brief introduction	191
5.3.1	The propagator structure in TFD	199
5.3.2	Non-hermitian representation of TFD	201
5.3.3	TFD for fields with continuous mass spectrum	202

5.4	The q -deformed Hopf algebra and the doubling of the field degrees of freedom	206
5.5	Free energy, entropy and the arrow of time. Intrinsic thermal nature of QFT	212
5.5.1	Entropy and system-environment entanglement	214
5.6	Thermal field theory and the gauge field	215
5.7	Boson condensation at finite temperature	221
5.7.1	Free energy and classical energy	225
5.8	Trajectories in the space of representations	231
6.	Selected topics in thermal field theory	235
6.1	Introduction	235
6.2	The Gell-Mann-Low formula and the closed time-path formalism	236
6.3	The functional integral approach	240
6.3.1	Generating functionals for Green's functions	240
6.3.2	The Feynman-Matthews-Salam formula	246
6.3.3	More on generating functionals	249
6.4	The effective action and the Schwinger-Dyson equations	252
6.5	Imaginary-time formalism	257
6.6	Geometric background for thermal field theories	260
6.6.1	The η - ξ spacetime	261
6.6.2	Fields in η - ξ spacetime	264
Appendix M	Thermal Wick theorem	271
Appendix N	Coherent state functional integrals	275
N.1	Glauber coherent states	275
N.2	Generalized coherent states	279
Appendix O	Imaginary-time formalism and phase transitions	287
O.1	Landau-Ginzburg treatment	290
Appendix P	Proof of Bogoliubov inequality	293
7.	Topological defects as non-homogeneous condensates. I	297
7.1	Introduction	297
7.2	Quantum field dynamics and classical soliton solutions	298

7.2.1	The dynamical map and the boson transformation	299
7.2.2	The quantum coordinate	302
7.3	The $\lambda\phi^4$ kink solution	304
7.3.1	The kink solution and temperature effects	309
7.3.2	The kink solution: closed time-path approach . .	312
7.4	The sine-Gordon solution	319
7.4.1	The quantum image of the Bäcklund transformations	324
7.5	Soliton solutions of the non-linear Schrödinger equation .	327
7.5.1	The ferromagnetic chain	328
7.5.2	Non-linear Schrödinger equation with Toda lattice back-reaction potential	334
7.5.3	Ring solitons in the Scheibe aggregates	336
7.6	Fermions in topologically non-trivial background fields .	339
7.7	Superfluid vortices	344
8.	Topological defects as non-homogeneous condensates. II	349
8.1	Introduction	349
8.2	Vortices in $U(1)$ local gauge theory	350
8.3	Topological solitons in gauge theories	353
8.3.1	Homogeneous boson condensation	357
8.3.2	The vortex of scalar electrodynamics	359
8.3.3	The 't Hooft-Polyakov monopole	362
8.3.4	The sphaleron	365
8.4	The $SU(2)$ instanton	367
9.	Dissipation and quantization	373
9.1	Introduction	373
9.2	The exact action for damped motion	374
9.2.1	Quantum Brownian motion	378
9.3	Quantum dissipation and unitarily inequivalent representations in QFT	379
9.3.1	The arrow of time and squeezed coherent states .	383
9.4	Dissipative non-commutative plane	385
9.4.1	The dissipative quantum phase interference . . .	388
9.5	Gauge structure and thermal features in particle mixing .	390
9.6	Dissipation and the many-body model of the brain	396
9.7	Quantization and dissipation	404

Appendix Q	Entropy and geometrical phases in neutrino mixing	413
Appendix R	Trajectories in the memory space	417
10.	Elements of soliton theory and related concepts	423
10.1	Introduction	423
10.2	The Korteweg–de Vries soliton	424
10.3	Topological solitons in $(1 + 1)$ -d relativistic field theories .	428
10.3.1	The sine-Gordon soliton	429
10.3.2	The $\lambda\phi^4$ kink	431
10.4	Topological solitons in gauge theories	434
10.4.1	The Nielsen–Olesen vortex	434
10.4.2	The ’t Hooft–Polyakov monopole	438
10.5	Topological defect classification and the Kibble–Zurek mechanism for defect formation	442
10.5.1	Exact homotopy sequences	445
10.5.2	Topological defects in theories with SSB	447
10.6	Derrick theorem and defect stability	453
10.7	Bogomol’nyi bounds	458
10.8	Non-topological solitons	463
10.9	Instantons and their manifestations	466
10.9.1	Collective coordinates and fermionic zero modes .	480
Appendix S	Bäcklund transformation for the sine-Gordon system	483
Appendix T	Elements of homotopy theory	487
	<i>Bibliography</i>	497
	<i>Index</i>	519