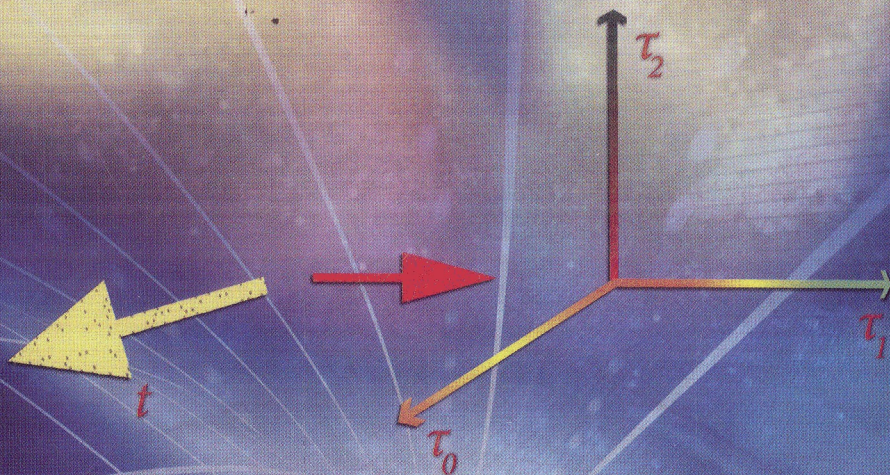
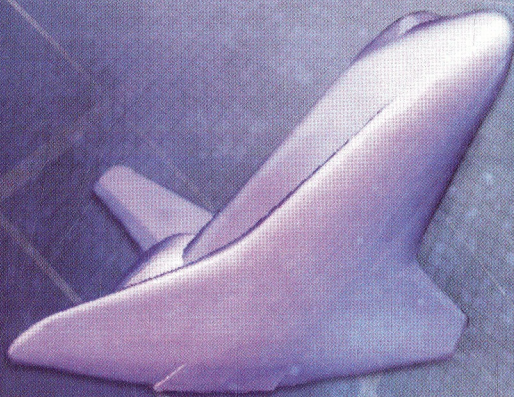


MULTIPLE SCALES THEORY AND AEROSPACE APPLICATIONS

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$$t \rightarrow \{\tau_i(t, \varepsilon)\} \quad \tau_i = \varepsilon \int k(t) dt$$



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