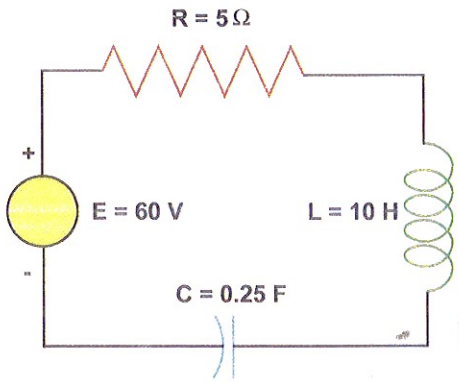
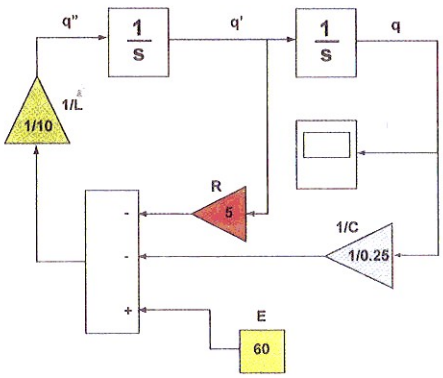
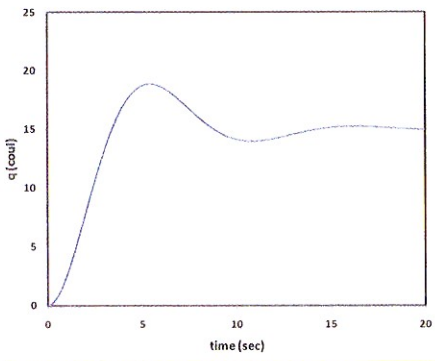


$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E$$





A First Course in Differential Equations, Modeling, and Simulation

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CRC Press
Taylor & Francis Group

$$q = 15 - e^{-0.25\,t}\left\{\frac{5\sqrt{15}}{3}\sin\left(\frac{3\sqrt{15}}{20}\,t\right) + 15\cos\left(\frac{3\sqrt{15}}{20}\,t\right)\right\}$$

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