

Contents

1 Puzzles, Patterns, and Mathematical Language 1

- 1.1 First Examples 1
- 1.2 Number Puzzles and Sequences 9
- 1.3 Truth-tellers, Liars, and Propositional Logic 24
- 1.4 Predicates 40
- 1.5 Implications 53
- 1.6 **Excursion** Validity of Arguments 68

2 A Primer of Mathematical Writing 81

- 2.1 Mathematical Writing 82
- 2.2 Proofs About Numbers 98
- 2.3 Mathematical Induction 110
- 2.4 More About Induction 122
- 2.5 Contradiction and the Pigeonhole Principle 132
- 2.6 **Excursion** Representations of Numbers 150
- 2.7 **Excursion** Modular Arithmetic and Cryptography 166

3 Sets and Boolean Algebra 181

- 3.1 Set Definitions and Operations 181
- 3.2 More Operations on Sets 198
- 3.3 Proving Set Properties 210
- 3.4 Boolean Algebra 221
- 3.5 **Excursion** Logic Circuits 229

4 Functions and Relations 248

- 4.1 Definitions, Diagrams, and Inverses 249
- 4.2 The Composition Operation 268
- 4.3 Properties of Functions and Set Cardinality 283
- 4.4 Properties of Relations 301
- 4.5 Equivalence Relations 313
- 4.6 Numerical Functions in Discrete Math 324
- 4.7 **Excursion** Iterated Functions and Chaos 334
- 4.8 **Excursion** Growth of Functions 345

5 Combinatorics 368

- 5.1 Introduction 369
- 5.2 Basic Rules for Counting 386
- 5.3 Combinations and the Binomial Theorem 398
- 5.4 Binary Sequences 408
- 5.5 Recursive Counting 418
- 5.6 **Excursion** Solving Recurrence Relations 423

6 Probability 440

- 6.1 Introduction 440
- 6.2 Sum and Product Rules for Probability 448
- 6.3 Probability in Games of Chance 460
- 6.4 Expected Value in Games of Chance 466
- 6.5 **Excursion** Recursion Revisited 475
- 6.6 **Excursion** Matrices and Markov Chains 482

7 Graphs and Trees 505

- 7.1 Graph Theory 506
- 7.2 Proofs About Graphs and Trees 519
- 7.3 Isomorphism and Planarity 533
- 7.4 Connections to Matrices and Relations 546
- 7.5 Graphs in Puzzles and Games 567
- 7.6 **Excursion** Binary Trees 581
- 7.7 **Excursion** Hamiltonian Cycles and the TSP 596

A Rules of the Game 613

- Cards 613
- Sports 614
- Miscellaneous Games 615

B Matrices and Their Operations 618

- Matrix Operations 618
- Matrix Arithmetic with Technology 620

Selected Answers and Hints 625

References and Further Reading 682