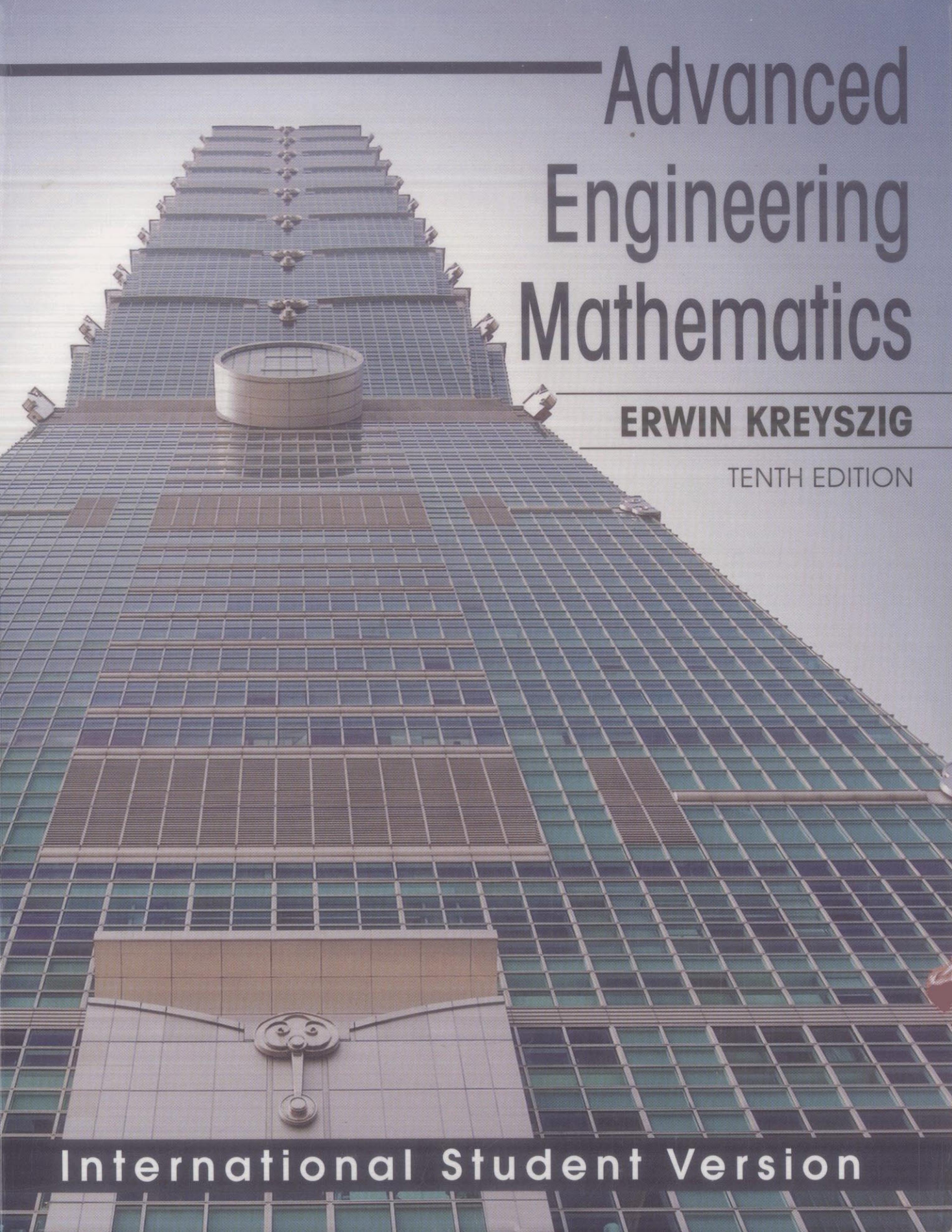

Advanced Engineering Mathematics

ERWIN KREYSZIG

TENTH EDITION

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