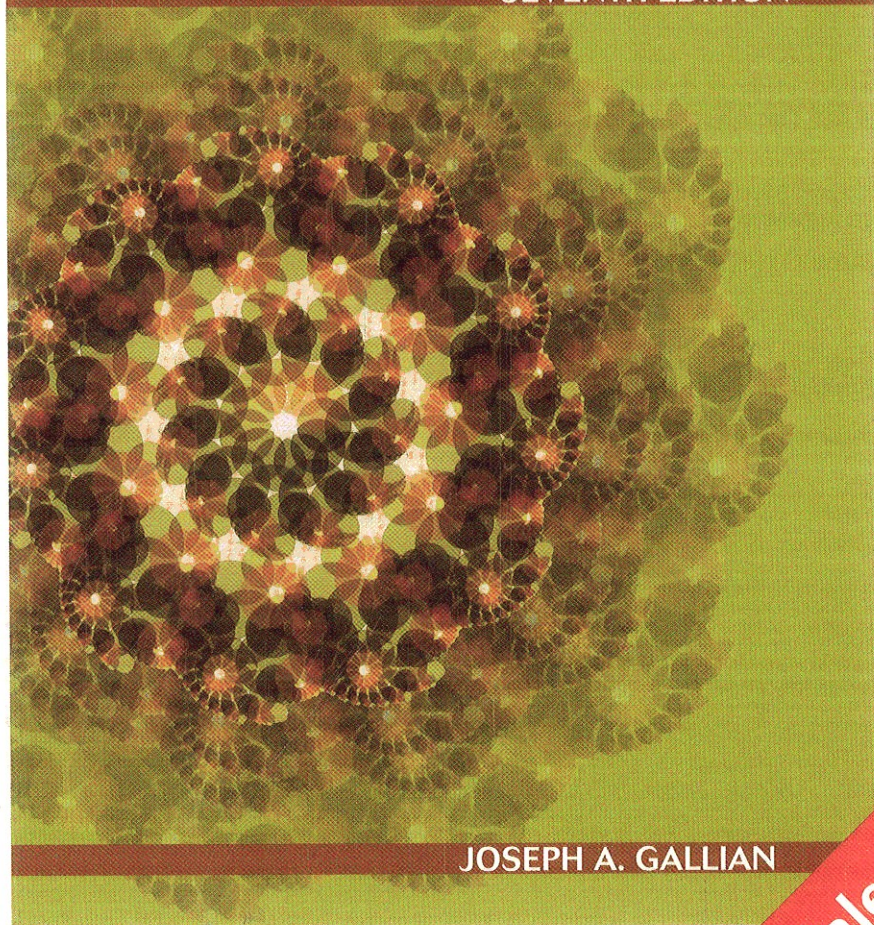


INTERNATIONAL EDITION

# CONTEMPORARY ABSTRACT ALGEBRA

SEVENTH EDITION



JOSEPH A. GALLIAN

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