

CONTENTS

Preface	ix
1. Introduction	1
1.1 Objectives of Legged Locomotion and Challenges in Controlling Dynamic Walking and Running	1
1.2 Literature Overview	4
1.2.1 Tracking of Time Trajectories	4
1.2.2 Poincaré Return Map and Hybrid Zero Dynamics	5
1.3 The Objective of the Book	7
1.3.1 Hybrid Zero Dynamics in Walking with Double Support Phase	7
1.3.2 Hybrid Zero Dynamics in Running with an Online Motion Planning Algorithm	8
1.3.3 Online Motion Planning Algorithms for Flight Phases of Running	9
1.3.4 Hybrid Zero Dynamics in 3D Running	10
1.3.5 Hybrid Zero Dynamics in Walking with Passive Knees	11
1.3.6 Hybrid Zero Dynamics with Continuous-Time Update Laws	12
2. Preliminaries in Hybrid Systems	13
2.1 Basic Definitions	13
2.2 Poincaré Return Map for Hybrid Systems	16
2.3 Low-Dimensional Stability Analysis	23
2.4 Stabilization Problem	28
3. Asymptotic Stabilization of Periodic Orbits for Walking with Double Support Phase	35
3.1 Introduction	35
3.2 Mechanical Model of a Biped Walker	37
3.2.1 The Biped Robot	37
3.2.2 Dynamics of the Flight Phase	37
3.2.3 Dynamics of the Single Support Phase	39
3.2.4 Dynamics of the Double Support Phase	40
3.2.5 Impact Model	43

3.2.6	Transition from the Double Support Phase to the Single Support Phase	45
3.2.7	Hybrid Model of Walking	45
3.3	Control Laws for the Single and Double Support Phases	46
3.3.1	Single Support Phase Control Law	46
3.3.2	Double Support Phase Control Law	49
3.4	Hybrid Zero Dynamics (HZD)	54
3.4.1	Analysis of HZD in the Single Support Phase	55
3.4.2	Analysis of HZD in the Double Support Phase	57
3.4.3	Restricted Poincaré Return Map	58
3.5	Design of an HZD Containing a Prespecified Periodic Solution	60
3.5.1	Design of the Output Functions	60
3.5.2	Design of u_{1d} and u_{2d}	62
3.6	Stabilization of the Periodic Orbit	67
3.7	Motion Planning Algorithm	71
3.7.1	Motion Planning Algorithm for the Single Support Phase	72
3.7.2	Motion Planning Algorithm for the Double Support Phase	73
3.7.3	Constructing a Period-One Orbit for the Open-Loop Hybrid Model of Walking	76
3.8	Numerical Example for the Motion Planning Algorithm	77
3.9	Simulation Results of the Closed-Loop Hybrid System	82
3.9.1	Effect of Double Support Phase on Angular Momentum Transfer and Stabilization	82
3.9.2	Effect of Event-Based Update Laws on Momentum Transfer and Stabilization	92
4.	Asymptotic Stabilization of Periodic Orbits for Planar Monopedal Running	95
4.1	Introduction	95
4.2	Mechanical Model of a Monopedal Runner	97
4.2.1	The Monopedal Runner	97
4.2.2	Dynamics of the Flight Phase	97
4.2.3	Dynamics of the Stance Phase	98
4.2.4	Open-Loop Hybrid Model of Running	99
4.3	Reconfiguration Algorithm for the Flight Phase	99
4.3.1	Determination of the Reachable Set	103
4.4	Control Laws for Stance and Flight Phases	120
4.4.1	Stance Phase Control Law	121
4.4.2	Flight Phase Control Law	122
4.4.3	Event-Based Update Law	124
4.5	Hybrid Zero Dynamics and Stabilization	125
4.6	Numerical Results	127

5. Online Generation of Joint Motions During Flight Phases of Planar Running	137
5.1 Introduction	137
5.2 Mechanical Model of a Planar Open Kinematic Chain	138
5.3 Motion Planning Algorithm to Generate Continuous Joint Motions	140
5.3.1 Determining the Reachable Set from the Origin	143
5.3.2 Motion Planning Algorithm	150
5.4 Motion Planning Algorithm to Generate Continuously Differentiable Joint Motions	152
6. Stabilization of Periodic Orbits for 3D Monopedal Running	159
6.1 Introduction	159
6.2 Open-Loop Hybrid Model of a 3D Running	160
6.2.1 Dynamics of the Flight Phase	162
6.2.2 Dynamics of the Stance Phase	163
6.2.3 Transition Maps	164
6.2.4 Hybrid Model	166
6.3 Design of a Period-One Solution for the Open-Loop Model of Running	167
6.4 Numerical Example	172
6.5 Within-Stride Controllers	175
6.5.1 Stance Phase Control Law	175
6.5.2 Flight Phase Control Law	178
6.6 Event-Based Update Laws for Hybrid Invariance	181
6.6.1 Takeoff Update Laws	184
6.6.2 Impact Update Laws	185
6.7 Stabilization Problem	186
6.8 Simulation Results	189
7. Stabilization of Periodic Orbits for Walking with Passive Knees	193
7.1 Introduction	193
7.2 Open-Loop Model of Walking	194
7.2.1 Mechanical Model of the Planar Bipedal Robot	194
7.2.2 Dynamics of the Single Support Phase	195
7.2.3 Impact Map	195
7.2.4 Open-Loop Impulsive Model of Walking	196
7.3 Motion Planning Algorithm	197
7.4 Numerical Example	200
7.5 Continuous-Times Controllers	202
7.6 Event-Based Controllers	209
7.6.1 Hybrid Invariance	209
7.6.2 Continuity of the Continuous-Time Controllers During the Within-Stride Transitions	212

CONTENTS

7.7	Stabilization Problem	213
7.8	Simulation of the Closed-Loop Hybrid System	217
8.	Continuous-Time Update Laws During Continuous Phases of Locomotion	221
8.1	Introduction	221
8.2	Invariance of the Exponential Stability Behavior for a Class of Impulsive Systems	222
8.3	Outline of the Proof of Theorem 8.1	224
8.4	Application to Legged Locomotion	227
A.	Proofs Associated with Chapter 3	229
A.1	Proof of Lemma 3.3	229
A.2	Proof of Lemma 3.4	230
A.3	Proof of Lemma 3.7	230
B.	Proofs Associated with Chapter 4	233
B.1	Proof of Lemma 4.2	233
B.2	Proof of Theorem 4.2	234
C.	Proofs Associated with Chapter 6	237
C.1	Proof of Lemma 6.1	237
C.2	Proof of Lemma 6.2	238
C.3	Invertibility of the Stance Phase Decoupling Matrix on the Periodic Orbit	240
	Bibliography	241
	Index	249