

Contents

1	Motivations and Scope	1
1.1	Introduction	1
1.2	Problem Statement	2
1.3	Finite Element Equations	4
1.4	Newton’s Method	6
1.5	The Line Search	7
1.6	Preconditioned Conjugate Gradients	10
1.7	Solved Problem	13
2	One-Dimensional Problem	15
2.1	Introduction	15
2.2	Elastoplastic Problem in One Dimension	15
2.3	Yield Function, Flow Rule, and Hardening/Softening	16
2.4	Loading/Unloading and Consistency Conditions	18
2.5	Isotropic and Kinematic Hardening	19
2.6	Energy and Plastic Dissipation	21
2.7	Some Properties of the Yield Function	23
2.8	Algorithm for Isotropic Hardening	24
2.9	Algorithm for Kinematic Hardening	26
2.10	Algorithm for Nonlinear Hardening	27
2.11	Uniqueness of Solution	28
2.12	Solved Problem	29
2.13	Summary	30
3	J2 Plasticity	31
3.1	Introduction	31
3.2	The J2 Yield Criterion	32
3.3	Perfect Plasticity	37
3.4	Radial Return Algorithm	38

3.5	Isotropic Hardening	40
3.6	Combined Isotropic-Kinematic Hardening	42
3.7	Algorithm for Combined Hardening	44
3.8	Algorithmic Tangent Operator	46
3.9	Maximum Plastic Dissipation	48
3.10	Non-associative Plasticity	51
3.11	Numerical Simulations	52
3.12	Summary	58
4	Isotropic Functions	59
4.1	Introduction	59
4.2	Spectral Representation	60
4.3	Spin of a Tensor	62
4.4	Constitutive Operators in Spectral Form	63
4.5	Lode's Angle	66
4.6	The Mohr-Coulomb Yield Criterion	69
4.7	Smooth Approximations of the MC Yield Surface	72
4.8	Pressure-Dependent Friction Angle	76
4.9	Flow Rule and Plastic Dilatancy	78
4.10	Cohesion and Friction Hardening	80
4.11	Return Mapping in Principal Axes	81
4.12	Algorithmic Tangent Operator	84
4.13	Return Mapping in Invariant Space	85
4.14	Numerical Example	87
4.15	Summary	89
5	Finite Deformation	91
5.1	Introduction	91
5.2	Basic Kinematics	91
5.3	Spectral Representation	94
5.4	Stress Tensors	97
5.5	Objectivity and Isotropy	97
5.6	Multiplicative Plasticity: Kinematics	101
5.7	Free Energy, Yield Function, and Plastic Flow Evolution	102
5.8	Elastoplastic Tangent Operator	104
5.9	Momentum Balance and Weak Form	107
5.10	Stress-Point Integration	109
5.11	Algorithmic Tangent Operator	112
5.12	Summary	115
6	Cap Models	117
6.1	Introduction	117
6.2	Infinitesimal Strain Hyperelasticity	119
6.3	Critical State Theory	121
6.4	Hyperelastic Law for MCC Model	125

6.5	Three-Invariant Formulation	127
6.6	Derivatives of Lode's Angle	128
6.7	A Plasticity Model for Sand	130
6.8	A Plasticity Model for Concrete	133
6.9	Return Mapping in Strain Space	135
6.10	Finite Deformation	138
6.11	Plane Strain Compression of a Sand	141
6.12	Summary	144
7	Discontinuities	145
7.1	Introduction	145
7.2	Contact with Cohesion and Friction	147
7.3	Lagrange Multipliers Method	150
7.4	Penalty and Augmented Lagrangian Methods	151
7.5	Enriched Finite Elements	154
7.6	Extended Finite Element Method	157
7.7	Stabilized Formulation	161
7.8	Strong Discontinuity	164
7.9	Assumed Enhanced Strain Method	168
7.10	Crack Tip Enrichment	172
7.11	Slip Weakening	175
7.12	Summary	177
8	Crystal Plasticity	179
8.1	Introduction	179
8.2	Kinematics of Crystal Slip	180
8.3	Constitutive Framework	182
8.4	Ultimate Algorithm	185
8.5	Uniqueness of Crystal Stress	188
8.6	Large Deformation	191
8.7	Multiscale Fields	195
8.8	Discrete Formulation	197
8.9	Stress-Point Integration	198
8.10	Multislip Systems	200
8.11	Twisting and Stretching of a Hollow Cylinder	205
8.12	Summary	206
9	Bifurcation	207
9.1	Introduction	207
9.2	Stability of Incrementally Linear Solids	208
9.3	Stability of Elastoplastic Solids	211
9.4	Deformation Bands	216
9.5	Spectral Representation	220
9.6	E-modes	224

9.7	Finite Deformation Bifurcation	227
9.8	Persistent Shear Band	231
9.9	Summary	233
References		235
Index		245