

Contents

Preface	xi
Prelude to Chapter 1	1
1 Introduction	3
1.1 What are Partial Differential Equations?	3
1.2 PDEs We Can Already Solve	6
1.3 Initial and Boundary Conditions	10
1.4 Linear PDEs—Definitions	12
1.5 Linear PDEs—The Principle of Superposition	16
1.6 Separation of Variables for Linear, Homogeneous PDEs	19
1.7 Eigenvalue Problems	25
Prelude to Chapter 2	41
2 The Big Three PDEs	43
2.1 Second-Order, Linear, Homogeneous PDEs with Constant Coefficients	43
2.2 The Heat Equation and Diffusion	44
2.3 The Wave Equation and the Vibrating String	54
2.4 Initial and Boundary Conditions for the Heat and Wave Equations	59
2.5 Laplace’s Equation—The Potential Equation	66
2.6 Using Separation of Variables to Solve the Big Three PDEs	71
Prelude to Chapter 3	77
3 Fourier Series	79
3.1 Introduction	79
3.2 Properties of Sine and Cosine	80
3.3 The Fourier Series	89
3.4 The Fourier Series, Continued	95
3.5 The Fourier Series—Proof of Pointwise Convergence	104
3.6 Fourier Sine and Cosine Series	117
3.7 Completeness	124

Prelude to Chapter 4	127
4 Solving the Big Three PDEs on Finite Domains	129
4.1 Solving the Homogeneous Heat Equation for a Finite Rod . . .	129
4.2 Solving the Homogeneous Wave Equation for a Finite String .	138
4.3 Solving the Homogeneous Laplace's Equation on a Rectangular Domain	147
4.4 Nonhomogeneous Problems	153
Prelude to Chapter 5	161
5 Characteristics	163
5.1 First-Order PDEs with Constant Coefficients	163
5.2 First-Order PDEs with Variable Coefficients	174
5.3 The Infinite String	180
5.4 Characteristics for Semi-Infinite and Finite String Problems .	192
5.5 General Second-Order Linear PDEs and Characteristics	201
Prelude to Chapter 6	211
6 Integral Transforms	213
6.1 The Laplace Transform for PDEs	213
6.2 Fourier Sine and Cosine Transforms	220
6.3 The Fourier Transform	230
6.4 The Infinite and Semi-Infinite Heat Equations	242
6.5 Distributions, the Dirac Delta Function and Generalized Fourier Transforms	254
6.6 Proof of the Fourier Integral Formula	266
Prelude to Chapter 7	275
7 Special Functions and Orthogonal Polynomials	277
7.1 The Special Functions and Their Differential Equations	277
7.2 Ordinary Points and Power Series Solutions; Chebyshev, Hermite and Legendre Polynomials	285
7.3 The Method of Frobenius; Laguerre Polynomials	292
7.4 Interlude: The Gamma Function	300
7.5 Bessel Functions	305
7.6 Recap: A List of Properties of Bessel Functions and Orthogonal Polynomials	317
Prelude to Chapter 8	327
8 Sturm–Liouville Theory and Generalized Fourier Series	329
8.1 Sturm–Liouville Problems	329
8.2 Regular and Periodic Sturm–Liouville Problems	337

8.3 Singular Sturm–Liouville Problems; Self-Adjoint Problems . . .	345
8.4 The Mean-Square or L^2 Norm and Convergence in the Mean .	354
8.5 Generalized Fourier Series; Parseval’s Equality and Completeness	361

Prelude to Chapter 9 **373**

9 PDEs in Higher Dimensions **375**

9.1 PDEs in Higher Dimensions: Examples and Derivations	375
9.2 The Heat and Wave Equations on a Rectangle; Multiple Fourier Series	386
9.3 Laplace’s Equation in Polar Coordinates: Poisson’s Integral Formula	402
9.4 The Wave and Heat Equations in Polar Coordinates	414
9.5 Problems in Spherical Coordinates	425
9.6 The Infinite Wave Equation and Multiple Fourier Transforms .	439
9.7 Postlude: Eigenvalues and Eigenfunctions of the Laplace Operator; Green’s Identities for the Laplacian	456

Prelude to Chapter 10 **463**

10 Nonhomogeneous Problems and Green’s Functions **465**

10.1 Green’s Functions for ODEs	465
10.2 Green’s Function and the Dirac Delta Function	484
10.3 Green’s Functions for Elliptic PDEs (I): Poisson’s Equation in Two Dimensions	500
10.4 Green’s Functions for Elliptic PDEs (II): Poisson’s Equation in Three Dimensions; the Helmholtz Equation	516
10.5 Green’s Functions for Equations of Evolution	525

Prelude to Chapter 11 **537**

11 Numerical Methods **539**

11.1 Finite Difference Approximations for ODEs	539
11.2 Finite Difference Approximations for PDEs	551
11.3 Spectral Methods and the Finite Element Method	565

A Uniform Convergence; Differentiation and Integration of Fourier Series **579**

B Other Important Theorems **585**

C Existence and Uniqueness Theorems **591**

D A Menagerie of PDEs **601**

E MATLAB Code for Figures and Exercises **613**

F	Answers to Selected Exercises	627
	References	647
	Index	655