

# Contents

<b>Preface</b>	<b>vii</b>
<b>Acknowledgments</b>	<b>xi</b>
<b>About the Author</b>	<b>xiii</b>
<b>1 Linear Programming</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.1.1 The Diet Problem . . . . .	1
1.1.2 Embedded Assumptions . . . . .	5
1.2 General Linear Programming Problems . . . . .	6
1.2.1 Standard Form of a Linear Program . . . . .	7
1.2.2 Linear Programming Terminology . . . . .	8
1.3 More Linear Programming Examples . . . . .	10
1.3.1 Converting Minimization Problems with Absolute Value	16
1.3.2 Network Optimization Models . . . . .	19
1.3.2.1 Minimum Cost Flow Problem . . . . .	20
1.3.2.2 Maximum Flow Problem . . . . .	23
1.3.3 Solving Linear Programs with MATLAB <sup>®</sup> . . . . .	25
1.4 Exercises . . . . .	29
1.5 Computational Project . . . . .	36
<b>2 Geometry of Linear Programming</b>	<b>43</b>
2.1 Introduction . . . . .	43
2.2 Geometry of the Feasible Set . . . . .	43
2.2.1 Geometry of Optimal Solutions . . . . .	49
2.3 Extreme Points and Basic Feasible Solutions . . . . .	52
2.3.1 Generating Basic Feasible Solutions . . . . .	58
2.3.1.1 Generating Basic Feasible Solutions with MATLAB <sup>®</sup> . . . . .	61
2.3.2 Degeneracy . . . . .	63
2.4 Resolution (Representation) Theorem . . . . .	64
2.4.1 Fundamental Theorem of Linear Programming . . . . .	67
2.5 Exercises . . . . .	69
	xv

<b>3</b>	<b>The Simplex Method</b>	<b>75</b>
3.1	Introduction	75
3.1.1	Example of Simplex Method	76
3.1.2	Adjacent Basic Feasible Solutions	77
3.2	Simplex Method Development	78
3.2.1	Checking Optimality of a Basic Feasible Solution	78
3.2.2	Moving to an Improved Adjacent Basic Feasible Solution	82
3.2.3	Simplex Method (Detailed Steps)	87
3.2.4	Finite Termination under Non-Degeneracy	101
3.3	Generating an Initial Basic Feasible Solution (Two-Phase and Big M Methods)	102
3.3.1	The Two-Phase Method	105
3.3.2	Big M Method	107
3.4	Degeneracy and Cycling	109
3.4.1	Anti-Cycling Rules (Bland's Rule and the Lexicographic Method)	110
3.4.1.1	Bland's Rule	110
3.4.1.2	Lexicographic (Perturbation) Method	112
3.5	Revised Simplex Method	118
3.5.1	Detailed Steps of the Revised Simplex Method	118
3.5.2	Advantages of the Revised Simplex Method	121
3.6	Complexity of the Simplex Method	122
3.7	Simplex Method MATLAB Code	123
3.7.1	MATLAB Code	124
3.8	Exercises	127
<b>4</b>	<b>Duality Theory</b>	<b>133</b>
4.1	Introduction	133
4.2	Motivation for Duality	133
4.3	Forming the Dual Problem for General Linear Programs	136
4.4	Weak and Strong Duality Theory	138
4.4.1	Primal-Dual Possibilities	144
4.5	Complementary Slackness	145
4.5.1	Complementary Slackness for Standard Form	148
4.6	Duality and the Simplex Method	150
4.6.1	Dual Simplex Method	153
4.7	Economic Interpretation of the Dual	160
4.7.1	Dual Variables and Marginal Values	160
4.8	Sensitivity Analysis	162
4.8.1	Changes in the Right-Hand Side Coefficients	163
4.8.2	Changes in the Cost (Objective) Coefficients	166
4.8.3	Changes in the Constraint Matrix	168
4.8.3.1	Adding a New Variable	168
4.8.3.2	Adding a New Constraint	169

4.9	Exercises . . . . .	172
<b>5</b>	<b>Dantzig-Wolfe Decomposition</b>	<b>177</b>
5.1	Introduction . . . . .	177
5.2	Decomposition for Block Angular Linear Programs . . . . .	177
5.3	Master Problem Reformulation . . . . .	180
5.4	Restricted Master Problem and the Revised Simplex Method	182
5.5	Dantzig-Wolfe Decomposition . . . . .	185
5.5.1	Economic Interpretation . . . . .	201
5.5.2	Initialization . . . . .	202
5.5.3	Bounds on Optimal Cost . . . . .	202
5.6	Dantzig-Wolfe MATLAB <sup>®</sup> Code . . . . .	205
5.6.1	DantzigWolfeDecomp MATLAB Code . . . . .	207
5.7	Exercises . . . . .	211
<b>6</b>	<b>Interior Point Methods</b>	<b>217</b>
6.1	Introduction . . . . .	217
6.2	Linear Programming Optimality Conditions . . . . .	217
6.2.1	Newton-Raphson Method . . . . .	219
6.3	Primal-Dual Interior Point Strategy . . . . .	223
6.3.1	Barrier Reformulation . . . . .	224
6.3.1.1	Newton-Raphson Method for KKT Conditions of the Barrier Problem . . . . .	225
6.3.2	General Primal-Dual Interior Point Method . . . . .	227
6.3.2.1	Starting with an Infeasible Interior Point . . . . .	228
6.3.3	Complexity of General Primal-Dual Interior Path Fol- lowing Methods . . . . .	229
6.3.3.1	Polynomial Complexity of Short-Step Path- Following Methods . . . . .	232
6.4	The Predictor-Corrector Variant of the Primal-Dual Interior Point Method . . . . .	233
6.4.1	Predictor Step . . . . .	233
6.4.2	Setting the Centering Parameter . . . . .	234
6.4.3	Corrector and Centering Step . . . . .	234
6.4.4	Computational Overhead . . . . .	235
6.4.5	Predictor-Corrector Algorithm . . . . .	235
6.4.5.1	Initial Point Generation . . . . .	236
6.5	Primal-Dual Interior Point Method in MATLAB <sup>®</sup> . . . . .	241
6.5.1	MATLAB Code . . . . .	242
6.6	Exercises . . . . .	244
<b>7</b>	<b>Quadratic Programming</b>	<b>249</b>
7.1	Introduction . . . . .	249
7.2	QP Model Structure . . . . .	249
7.3	QP Application: Financial Optimization . . . . .	251

7.4	Solving Quadratic Programs Using MATLAB®	257
7.4.1	Generating the Efficient Frontier Using MATLAB	258
7.5	Optimality Conditions for Quadratic Programming	259
7.5.1	Local and Global Optima	260
7.5.2	Unconstrained Quadratic Programs	261
7.5.2.1	Convex Unconstrained Quadratic Programming (Global Optimality)	265
7.5.3	Convex Optimization	268
7.5.4	Equality-Constrained Quadratic Programs	269
7.5.4.1	Alternative Solution Methods for EQP	273
7.5.5	Inequality Constrained Convex Quadratic Programming	274
7.5.6	Predictor-Corrector Algorithm for Convex QP	275
7.6	Exercises	281
<b>8</b>	<b>Linear Optimization under Uncertainty</b>	<b>287</b>
8.1	Introduction	287
8.2	Stochastic Programming	288
8.2.1	Stochastic Programming with Recourse	290
8.2.1.1	Two-Stage Stochastic Programming with Recourse	293
8.3	More Stochastic Programming Examples	297
8.3.1	Solving Two-Stage Stochastic Programs with Recourse	300
8.3.1.1	Dantzig-Wolfe Decomposition for Stochastic Programming	301
8.3.1.2	L-Shaped Method (Constraint Generation)	301
8.3.1.3	Convergence of the L-Shaped Method	309
8.4	Robust Optimization	310
8.4.1	Constraint-wise Construction of $RC$	311
8.5	Exercises	318
<b>A</b>	<b>Linear Algebra Review</b>	<b>323</b>
	<b>Bibliography</b>	<b>329</b>
	<b>Index</b>	<b>337</b>