Contents

Preface *page* xii
Acknowledgments xx
Notation xxi

1 **Signals and systems** 1
1.1 Continuous and discrete signals 1
1.2 Unit step and nascent delta functions 4
1.3 Relationship between complex exponentials and delta functions 7
1.4 Attributes of signals 9
1.5 Signal arithmetics and transformations 11
1.6 Linear and time-invariant systems 15
1.7 Signals through continuous LTI systems 17
1.8 Signals through discrete LTI systems 21
1.9 Continuous and discrete convolutions 24
1.10 Homework problems 29

2 **Vector spaces and signal representation** 34
2.1 Inner product space 34
2.1.1 Vector space 34
2.1.2 Inner product space 36
2.1.3 Bases of vector space 43
2.1.4 Signal representation by orthogonal bases 47
2.1.5 Signal representation by standard bases 52
2.1.6 An example: the Fourier transforms 55
2.2 Unitary transformation and signal representation 57
2.2.1 Linear transformation 57
2.2.2 Eigenvalue problems 59
2.2.3 Eigenvectors of D^2 as Fourier basis 61
2.2.4 Unitary transformations 64
2.2.5 Unitary transformations in N-D space 66
2.3 Projection theorem and signal approximation 70
2.3.1 Projection theorem and pseudo-inverse 70

2.3.2 Signal approximation 76
2.4 Frames and biorthogonal bases 81
2.4.1 Frames 81
2.4.2 Signal expansion by frames and Riesz bases 82
2.4.3 Frames in finite-dimensional space 90
2.5 Kernel function and Mercer's theorem 93
2.6 Summary 99
2.7 Homework problems 101

3 Continuous-time Fourier transform 105
3.1 The Fourier series expansion of periodic signals 105
3.1.1 Formulation of the Fourier expansion 105
3.1.2 Physical interpretation 107
3.1.3 Properties of the Fourier series expansion 109
3.1.4 The Fourier expansion of typical functions 111
3.2 The Fourier transform of non-periodic signals 119
3.2.1 Formulation of the CTFT 119
3.2.2 Relation to the Fourier expansion 124
3.2.3 Properties of the Fourier transform 125
3.2.4 Fourier spectra of typical functions 132
3.2.5 The uncertainty principle 140
3.3 Homework problems 142

4 Discrete-time Fourier transform 146
4.1 Discrete-time Fourier transform 146
4.1.1 Fourier transform of discrete signals 146
4.1.2 Properties of the DTFT 151
4.1.3 DTFT of typical functions 157
4.1.4 The sampling theorem 160
4.1.5 Reconstruction by interpolation 170
4.2 Discrete Fourier transform 173
4.2.1 Formulation of the DFT 173
4.2.2 Array representation 179
4.2.3 Properties of the DFT 183
4.2.4 Four different forms of the Fourier transform 192
4.2.5 DFT computation and fast Fourier transform 196
4.3 Two-dimensional Fourier transform 201
4.3.1 Two-dimensional signals and their spectra 201
4.3.2 Fourier transform of typical 2-D functions 204
4.3.3 Four forms of 2-D Fourier transform 207
4.3.4 Computation of the 2-D DFT 209
4.4 Homework problems 215

5 Applications of the Fourier transforms 220
5.1 LTI systems in time and frequency domains 220
5.2 Solving differential and difference equations 225
5.3 Magnitude and phase filtering 232
5.4 Implementation of 1-D filtering 238
5.5 Implementation of 2-D filtering 249
5.6 Hilbert transform and analytic signals 256
5.7 Radon transform and image restoration from projections 261
5.8 Orthogonal frequency-division modulation (OFDM) 269
5.9 Homework problems 271

6 The Laplace and z-transforms 277
6.1 The Laplace transform 277
6.1.1 From Fourier transform to Laplace transform 277
6.1.2 The region of convergence 280
6.1.3 Properties of the Laplace transform 281
6.1.4 The Laplace transform of typical signals 284
6.1.5 Analysis of continuous LTI systems by Laplace transform 286
6.1.6 First-order system 292
6.1.7 Second-order system 295
6.1.8 The unilateral Laplace transform 307
6.2 The z-transform 311
6.2.1 From Fourier transform to z-transform 311
6.2.2 Region of convergence 314
6.2.3 Properties of the z-transform 316
6.2.4 The z-transform of typical signals 321
6.2.5 Analysis of discrete LTI systems by z-transform 322
6.2.6 First- and second-order systems 327
6.2.7 The unilateral z-transform 332
6.3 Homework problems 335

7 Fourier-related orthogonal transforms 339
7.1 The Hartley transform 339
7.1.1 Continuous Hartley transform 339
7.1.2 Properties of the Hartley transform 341
7.1.3 Hartley transform of typical signals 343
7.1.4 Discrete Hartley transform 345
7.1.5 The 2-D Hartley transform 348
7.2 The discrete sine and cosine transforms 353
7.2.1 The continuous cosine and sine transforms 353
7.2.2 From DFT to DCT and DST 355
7.2.3 Matrix forms of DCT and DST 360
7.2.4 Fast algorithms for the DCT and DST 366

7.2.5 DCT and DST filtering 370
7.2.6 The 2-D DCT and DST 373
7.3 Homework problems 377

8 The Walsh-Hadamard, slant, and Haar transforms 379
8.1 The Walsh-Hadamard transform 379
8.1.1 Hadamard matrix 379
8.1.2 Hadamard-ordered Walsh-Hadamard transform ($\mathrm{WHT_h}$) 381
8.1.3 Fast Walsh-Hadamard transform algorithm 382
8.1.4 Sequency-ordered Walsh-Hadamard matrix ($\mathrm{WHT_w}$) 384
8.1.5 Fast Walsh-Hadamard transform (sequency ordered) 386
8.2 The slant transform 392
8.2.1 Slant matrix 392
8.2.2 Slant transform and its fast algorithm 395
8.3 The Haar transform 398
8.3.1 Continuous Haar transform 398
8.3.2 Discrete Haar transform 400
8.3.3 Computation of the discrete Haar transform 403
8.3.4 Filter bank implementation 405
8.4 Two-dimensional transforms 408
8.5 Homework problems 411

9 Karhunen-Loève transform and principal component analysis 412
9.1 Stochastic process and signal correlation 412
9.1.1 Signals as stochastic processes 412
9.1.2 Signal correlation 415
9.2 Karhunen-Loève transform (KLT) 417
9.2.1 Continuous KLT 417
9.2.2 Discrete KLT 418
9.2.3 Optimalities of the KLT 419
9.2.4 Geometric interpretation of the KLT 423
9.2.5 Principal component analysis (PCA) 426
9.2.6 Comparison with other orthogonal transforms 427
9.2.7 Approximation of the KLT by the DCT 432
9.3 Applications of the KLT 438
9.3.1 Image processing and analysis 438
9.3.2 Feature extraction for pattern classification 444
9.4 Singular value decomposition transform 449
9.4.1 Singular value decomposition 449
9.4.2 Application in image compression 454
9.5 Homework problems 456

10 Continuous- and discrete-time wavelet transforms 461

10.1 Why wavelet? 461
10.1.1 Short-time Fourier transform and Gabor transform 461
10.1.2 The Heisenberg uncertainty 462
10.2 Continuous-time wavelet transform (CTWT) 464
10.2.1 Mother and daughter wavelets 464
10.2.2 The forward and inverse wavelet transforms 466
10.3 Properties of the CTWT 468
10.4 Typical mother wavelet functions 471
10.5 Discrete-time wavelet transform (DTWT) 474
10.5.1 Discretization of wavelet functions 474
10.5.2 The forward and inverse transform 476
10.5.3 A fast inverse transform algorithm 478
10.6 Wavelet transform computation 481
10.7 Filtering based on wavelet transform 484
10.8 Homework problems 490

11 Multiresolution analysis and discrete wavelet transform 492
11.1 Multiresolution analysis (MRA) 492
11.1.1 Scale spaces 492
11.1.2 Wavelet spaces 498
11.1.3 Properties of the scaling and wavelet filters 501
11.1.4 Relationship between scaling and wavelet filters 504
11.1.5 Wavelet series expansion 506
11.1.6 Construction of scaling and wavelet functions 508
11.2 Discrete wavelet transform (DWT) 518
11.2.1 Discrete wavelet transform (DWT) 518
11.2.2 Fast wavelet transform (FWT) 521
11.3 Filter bank implementation of DWT and inverse DWT 523
11.3.1 Two-channel filter bank and inverse DWT 523
11.3.2 Two-dimensional DWT 530
11.4 Applications in filtering and compression 535
11.5 Homework problems 542

Appendices 546

A Review of linear algebra 546
A.1 Basic definitions 546
A.2 Eigenvalues and eigenvectors 551
A.3 Hermitian matrix and unitary matrix 552
A.4 Toeplitz and circulant matrices 554
A.5 Vector and matrix differentiation 554

B Review of random variables 556

B.1 Random variables 556
B.2 Multivariate random variables 558
B.3 Stochastic models 562

Bibliography 565
Index 566