Contents

	Preface	xv
1	**Introduction**	1
1.1	Some Apparently Simple Questions	1
1.2	An Alternative Analytic Framework	3
	Exercises	4
2	**Linear Equations and Computer Basics**	7
2.1	L-U Factorization	8
2.2	Gaussian Elimination	10
2.3	Rounding Error	12
2.4	Ill Conditioning	13
2.5	Special Linear Equations	15
2.6	Iterative Methods	16
	Exercises	19
	Appendix 2A: Computer Arithmetic	20
	Appendix 2B: Data Storage	24
	Bibliographic Notes	26
3	**Nonlinear Equations and Complementarity Problems**	29
3.1	Bisection Method	30
3.2	Function Iteration	32
3.3	Newton's Method	33
3.4	Quasi-Newton Methods	36
3.5	Problems with Newton Methods	40
3.6	Choosing a Solution Method	42
3.7	Complementarity Problems	44
3.8	Complementarity Methods	47
	Exercises	52
	Bibliographic Notes	57
4	**Finite-Dimensional Optimization**	59
4.1	Derivative-Free Methods	60
4.2	Newton-Raphson Method	65
4.3	Quasi-Newton Methods	66
4.4	Line Search Methods	70

4.5	Special Cases	72
4.6	Constrained Optimization	74
	Exercises	78
	Bibliographic Notes	83
5	**Numerical Integration and Differentiation**	85
5.1	Newton-Cotes Methods	85
5.2	Gaussian Quadrature	88
5.3	Monte Carlo Integration	90
5.4	Quasi–Monte Carlo Integration	92
5.5	An Integration Tool Kit	94
5.6	Numerical Differentiation	97
5.7	Initial Value Problems	105
	Exercises	110
	Bibliographic Notes	114
6	**Function Approximation**	115
6.1	Interpolation Principles	116
6.2	Polynomial Interpolation	118
6.3	Piecewise Polynomial Splines	123
6.4	Piecewise Linear Basis Functions	129
6.5	Multidimensional Interpolation	130
6.6	Choosing an Approximation Method	134
6.7	An Approximation Tool Kit	135
6.8	The Collocation Method	141
6.9	Boundary Value Problems	146
	Exercises	149
	Bibliographic Notes	152
7	**Discrete Time, Discrete State Dynamic Models**	155
7.1	Discrete Dynamic Programming	155
7.2	Economic Examples	157
	7.2.1 Mine Management	157
	7.2.2 Asset Replacement	158
	7.2.3 Asset Replacement with Maintenance	159

7.2.4 Option Pricing 160
7.2.5 Water Management 161
7.2.6 Bioeconomic Model 162
7.3 Solution Algorithms 163
7.3.1 Backward Recursion 164
7.3.2 Function Iteration 165
7.3.3 Policy Iteration 165
7.3.4 Curse of Dimensionality 166
7.4 Dynamic Simulation Analysis 167
7.5 A Discrete Dynamic Programming Tool Kit 169
7.6 Numerical Examples 172
7.6.1 Mine Management 172
7.6.2 Asset Replacement 175
7.6.3 Asset Replacement with Maintenance 176
7.6.4 Option Pricing 178
7.6.5 Water Management 180
7.6.6 Bioeconomic Model 182
Exercises 185
Bibliographic Notes 188

8 Discrete Time, Continuous State Dynamic Models: Theory and Examples 189
8.1 Continuous State Dynamic Programming 190
8.2 Euler Conditions 191
8.3 Continuous State, Discrete Choice Models 194
8.3.1 Asset Replacement 194
8.3.2 Industry Entry and Exit 195
8.3.3 American Option Pricing 196
8.4 Continuous State, Continuous Choice Models 197
8.4.1 Economic Growth 197
8.4.2 Renewable Resource Management 198
8.4.3 Nonrenewable Resource Management 200
8.4.4 Water Management 201
8.4.5 Monetary Policy 202
8.4.6 Production-Adjustment Model 204

	8.4.7 Production-Inventory Model	205
	8.4.8 Livestock Feeding	207
8.5	Dynamic Games	208
	8.5.1 Capital-Production Game	209
	8.5.2 Income Redistribution Game	210
	8.5.3 Marketing Board Game	211
8.6	Rational Expectations Models	212
	8.6.1 Asset Pricing Model	214
	8.6.2 Competitive Storage	215
	8.6.3 Government Price Controls	217
	Exercises	218
	Bibliographic Notes	221
9	**Discrete Time, Continuous State Dynamic Models: Methods**	223
9.1	Linear-Quadratic Control	223
9.2	Bellman Equation Collocation Methods	227
9.3	Implementation of the Collocation Method	230
9.4	A Continuous State Dynamic Programming Tool Kit	237
9.5	Postoptimality Analysis	238
9.6	Computational Examples: Discrete Choice	240
	9.6.1 Asset Replacement	240
	9.6.2 Industry Entry and Exit	243
9.7	Computational Examples: Continuous Choice	246
	9.7.1 Economic Growth	246
	9.7.2 Renewable Resource Management	250
	9.7.3 Nonrenewable Resource Management	253
	9.7.4 Water Management	256
	9.7.5 Monetary Policy	259
	9.7.6 Production-Adjustment Model	264
	9.7.7 Production-Inventory Model	266
	9.7.8 Livestock Feeding	271
9.8	Dynamic Game Methods	273
	9.8.1 Capital-Production Game	279
	9.8.2 Income Redistribution Game	283
	9.8.3 Marketing Board Game	286

9.9 Rational Expectations Methods 291
9.9.1 Asset Pricing Model 295
9.9.2 Competitive Storage 298
9.9.3 Government Price Controls 302
Exercises 306
Bibliographic Notes 309

10 Continuous Time Models: Theory and Examples 311
10.1 Arbitrage-Based Asset Valuation 311
10.1.1 Bond Pricing 314
10.1.2 Black-Scholes Option Pricing Formula 315
10.1.3 Stochastic Volatility Model 316
10.1.4 Exotic Options 317
10.1.5 Multivariate Affine Asset Pricing Model 319
10.2 Continuous Action Control 320
10.2.1 Choice of the Discount Rate 324
10.2.2 Euler Equation Methods 325
10.2.3 Bang-Bang Problems 327
10.3 Continuous Action Control Examples 328
10.3.1 Nonrenewable Resource Management 328
10.3.2 Neoclassical Growth Model 329
10.3.3 Optimal Renewable Resource Extraction 330
10.3.4 Stochastic Growth 332
10.3.5 Portfolio Choice 333
10.3.6 Production with Adjustment Costs 336
10.3.7 Harvesting a Renewable Resource 337
10.3.8 Sequential Learning 338
10.4 Regime Switching Methods 342
10.4.1 Machine Abandonment 343
10.4.2 American Put Option 345
10.4.3 Entry-Exit 345
10.5 Impulse Control 347
10.5.1 Asset Replacement 354
10.5.2 Timber Harvesting 355
10.5.3 Storage Management 356

10.5.4 Capacity Choice 356
10.5.5 Cash Management 357
Exercises 358
Appendix 10A: Dynamic Programming and Optimal Control Theory 367
Bibliographic Notes 368

11 Continuous Time Models: Solution Methods 371
11.1 Solving Arbitrage Valuation Problems 372
11.1.1 A Simple Bond Pricing Model 373
11.1.2 More General Assets 375
11.1.3 An Asset Pricing Solver 379
11.1.4 Black-Scholes Option Pricing Formula 382
11.1.5 Stochastic Volatility Model 385
11.1.6 American Options 387
11.1.7 Exotic Options 391
11.1.8 Affine Asset Pricing Models 400
11.1.9 Calibration 401
11.2 Solving Stochastic Control Problems 405
11.2.1 A Solver for Stochastic Control Problems 406
11.2.2 Postoptimality Analysis 409
11.3 Stochastic Control Examples 412
11.3.1 Optimal Growth 412
11.3.2 Renewable Resource Management 415
11.3.3 Production with Adjustment Costs 417
11.3.4 Optimal Fish Harvest 420
11.3.5 Sequential Learning 423
11.4 Regime Switching Models 428
11.4.1 Asset Abandonment 431
11.4.2 Optimal Fish Harvest 432
11.4.3 Entry-Exit 434
11.5 Impulse Control 437
11.5.1 Asset Replacement 440
11.5.2 Timber Management 441
11.5.3 Storage Management 443
11.5.4 Cash Management 455

11.5.5 Optimal Fish Harvest 456
Exercises 450
Appendix 11 A: Basis Matrices for Multivariate Models 465
Bibliographic Notes 457

Appendix A: Mathematical Background 459
A.1 Normed Linear Spaces 459
A.2 Matrix Algebra 462
A.3 Real Analysis 464
A.4 Markov Chains 465
A.5 Continuous Time Mathematics 467
A.5.1 Ito Processes 467
A.5.2 Forward and Backward Equations 470
A.5.3 The Feynman-Kac Equation 473
A.5.4 Geometric Brownian Motion 475
Bibliographic Notes 476

Appendix B: A MATLAB Primer 477
B.1 The Basics 477
B.2 Conditional Statements and Looping 481
B.3 Scripts and Functions 483
B.4 Debugging 488
B.5 Other Data Types 490
B.6 Programming Style 491

References 493
Index 499