Contents

Pι	reface			page xv
1	Radiat	ion and i	nitial-value problems for the wave equation	1
	1.1	The rad	diation problem	1
		1.1.1	Fourier integral representations	2
	1.2	Green	functions	6
		1.2.1	Retarded and advanced Green functions	7
		1.2.2	Frequency-domain Green functions	9
	1.3	Green-	function solutions to the radiation problem	12
		1.3.1	The primary field solution	14
		1.3.2	Representation of the radiated field in terms of boundary	
			values via the Kirchhoff-Helmholtz theorem	16
		1.3.3	The interior field solution	18
	1.4	The in	itial-value problem for the wave equation	20
		1.4.1	Uniqueness	21
		1.4.2	Field back propagation	21
	1.5	Freque	ency-domain solution of the radiation problem	22
		1.5.1	The radiation pattern and the Sommerfeld	
			condition	23
	1.6	Radiat	ed power and energy	25
	1.7	Non-ra	adiating sources	27
		1.7.1	Non-radiating sources in the frequency domain	29
		1.7.2	A source decomposition theorem	31
		1.7.3	Essentially non-radiating sources	33
		1.7.4	The field uniqueness theorem	35
	1.8	Surfac	e sources	36
		1.8.1	Non-radiating surface sources	38
		1.8.2	Active object cloaking	39
	Furth	er readin	ng	40
	Probl	ems		41
2	Radiat	tion and b	ooundary-value problems in the frequency domain	43
	2.1	Freque	ency-domain formulation of the radiation problem	43
		2.1.1	Analytic-signal representation of time-domain fields	44
		2.1.2	The Helmholtz equation	45
		2.1.3	Lorentz dispersive medium	45

	2.1.4	The Sommerfeld radiation condition in dispersive media	47
	2.1.5	Incoming- and conjugate-wave radiation conditions	48
2.2	Green f	functions	50
	2.2.1	Green functions in two space dimensions	52
2.3	Time-d	omain Green functions	53
	2.3.1	Key features of the time-domain Green functions	54
2.4	Green-f	function solution of the radiation problem	55
	2.4.1	Solution of the radiation problem in two space dimensions	57
2.5	The Ki	rchhoff-Helmholtz representation of the radiated field	58
	2.5.1	The interior field solution and field back propagation	59
2.6	Radiate	ed power and energy	60
2.7	Non-ra	diating and essentially non-radiating sources in dispersive media	63
	2.7.1	Non-radiating sources and the radiation pattern	63
	2.7.2	Essentially non-radiating sources	64
2.8	Bounda	ary-value problems for the Helmholtz equation	65
	2.8.1	The interior boundary-value problem	67
	2.8.2	The exterior boundary-value problem for closed boundaries	68
	2.8.3	The exterior boundary-value problem for open boundaries	70
	2.8.4	Symmetry of the Green functions	71
2.9	The Ra	yleigh–Sommerfeld boundary-value problem	72
	2.9.1	The Rayleigh-Sommerfeld solution for two-dimensional	
		wavefields	76
	2.9.2	Rayleigh-Sommerfeld representation of the radiated field	76
2.10	Solutio	n of the RS problem using the Helmholtz identities	77
2.11	Back p	ropagation and the inverse RS boundary-value problem	78
	2.11.1	The inverse RS boundary-value problem	79
	2.11.2	Connection with wavefield time reversal	80
2.12	Surface	e sources in dispersive media	81
	2.12.1	Non-radiating surface sources	82
Furth	er readin	g	84
Probl	ems		85
Eigenf	unction e	xpansions of solutions to the Helmholtz equation	87
3.1	Separat	tion of variables and the Sturm-Liouville problem	87
	3.1.1	The Sturm–Liouville problem	88
3.2	Cartesi	an coordinates	89
	3.2.1	Homogeneous plane-wave expansions	90
	3.2.2	Plane-wave expansions that include inhomogeneous plane waves	92
	3.2.3	Plane-wave expansions involving evanescent plane waves	94
3.3	Spheric	cal coordinates	99
3.4	Multip	ole expansions	104
	3.4.1	Multipole expansions of the Dirichlet and Neumann Green	
		functions	107
	3.4.2	Plane-wave expansions of the multipole fields	110

	3.5		r cylindrical coordinates	111
	3.6	Two-di	mensional wavefields	113
		3.6.1	Polar coordinates	113
	Furth	er reading		116
	Proble	ems		116
4	Angula	ar-spectru	m and multipole expansions	118
	4.1	The We	eyl expansion	118
		4.1.1	The angular-spectrum expansion for the conjugate-wave	
			Green function	120
		4.1.2	The angular-spectrum expansion of the incoming-wave Green function	121
		4.1.3	Angle-variable forms of the Green-function expansions	122
	4.2		gular-spectrum expansion of the radiated field	125
		4.2.1	The angle-variable form of the radiated field expansion	127
		4.2.2	The angular spectrum and radiation pattern	127
		4.2.3	The radiation pattern of a non-radiating source	129
	4.3		d and back propagation using the angular spectrum	129
		4.3.1	Back propagation from the radiation pattern	133
	4.4	Stabiliz	ted field back propagation and the inverse boundary-value problem	133
		4.4.1	Back propagation using the incoming-wave Green function	134
		4.4.2	Back propagation using the conjugate-wave Green function	
			and field time reversal	136
	4.5	The ang	gular-spectrum expansion of the scalar wavelet field	137
	4.6	Angula	r-spectrum expansions in two space dimensions	139
		4.6.1	The angular-spectrum expansion of the solution to the 2D	
			radiation problem	141
		4.6.2	Two-dimensional forward and back propagation	142
		4.6.3	The angle-variable form of the 2D angular-spectrum expansion	146
	4.7	The Fre	esnel approximation and Fresnel transform	147
		4.7.1	The 3D Fresnel approximation and Fresnel transform	148
		4.7.2	The 2D Fresnel approximation	151
	4.8	Multipo	ole expansions	153
		4.8.1	Multipole expansion of the radiated field	154
		4.8.2	Forward and back propagation using the multipole expansion	155
		4.8.3	Back propagation in the interior boundary-value problem	157
		4.8.4	Back propagation from the radiation pattern	158
	4.9	Multipo	ole expansions of two-dimensional wavefields	160
	4.10	Connec	ction between the angular-spectrum and multipole expansions	161
	4.11	Radiate	ed energy out of plane and spherical boundaries	163
		4.11.1	Radiated energy into an infinite half-space	164
		4.11.2	Radiated energy from a spherical region	166
	Furth	er readin	g	167
	Probl	ems		167

5	The in	verse soui	rce problem	169			
	5.1	The IS	P for the wave equation	169			
		5.1.1	The ISP integral equation	171			
		5.1.2	The Porter–Bojarski integral equation	174			
		5.1.3	Time reversal and the back-propagated field	176			
		5.1.4	The ISP in terms of Dirichlet or Neumann boundary-value data	177			
	5.2	The IS	P for surface sources	179			
		5.2.1	The ISP for a planar surface source	179			
		5.2.2	Solving the ISP integral equation	180			
		5.2.3	Interpretation of the solution	182			
	5.3	The IS	P for 3D sources supported in plane-parallel slabs	183			
		5.3.1	Solving for the source	184			
		5.3.2	Limiting form as a surface source	186			
		5.3.3	Time-reversal imaging for slab geometry	187			
	5.4	The Hi	ilbert-space formulation of the ISP	188			
		5.4.1	The adjoint operator	191			
		5.4.2	Singular value decomposition	194			
		5.4.3	The range and null space of \hat{T}	197			
		5.4.4	The least-squares pseudo-inverse	198			
		5.4.5	Filtered back propagation and back-propagation imaging	201			
	5.5	The an	itenna-synthesis problem	202			
		5.5.1	Implementation of the SVD	203			
		5.5.2	The solution to the far-field ISP	206			
		5.5.3	The algorithm point-spread function	209			
		5.5.4	Time reversal and back-propagation imaging	210			
	5.6	Picard'	's condition and minimum-sized sources	211			
	5.7	Antenr	na synthesis and the far-field ISP in 2D space	214			
		5.7.1	Implementation of the SVD	215			
		5.7.2	The solution to the 2D far-field ISP and algorithm PSF	217			
		5.7.3	Two-dimensional scalar wavelet source	220			
	5.8	The lin	nited-view problem	222			
		5.8.1	The 2D limited-view problem	223			
		5.8.2	Computing the singular system	224			
	Furth	er readin	ng	225			
	Probl	ems		225			
6	Scattering theory						
	6.1						
	6.2		ippmann–Schwinger equation	230 232			
		6.2.1	The Lippmann–Schwinger equation for the full Green function	233			
		6.2.2	The formal solution to the LS equation	234			
	6.3		ring from homogeneous penetrable objects	235			
		6.3.1	Scattering from homogeneous spheres and cylinders	236			
		632	Scattering from a homogeneous sphere	236			

	6.3.3	Scattering from a homogeneous cylinder	238
	6.3.4	Scattering from concentric cylinders	239
6.4	The sca	ttering amplitude	240
	6.4.1	The scattering amplitude in 2D space	242
	6.4.2	Reciprocity and translation theorems for the scattering amplitude	244
	6.4.3	Scattered field energy and the optical theorem	245
6.5	Comput	ting the scattered field from the scattering amplitude	246
	6.5.1	Field scattered by an arbitrary incident wave and the general-	
		ized scattering amplitude	247
	6.5.2	Computing the scattering amplitude from scattered field data	
		over a plane	248
	6.5.3	Multipole expansion of the scattered field	249
	6.5.4	Multipole expansions of 2D scattered fields	251
6.6	The tran	nsition operator	253
	6.6.1	The Lippmann–Schwinger equation for the transition operator	254
6.7	The Bo	rn series	255
	6.7.1	The Born approximation	256
	6.7.2	Incident plane waves	256
6.8	The Bo	rn approximation for spherically and cylindrically symmetric	
	scatterii	ng potentials	259
	6.8.1	Born scattering from homogeneous cylinders	261
	6.8.2	The error between the Born and exact scattering amplitudes	262
6.9	Non-sca	attering potentials	265
	6.9.1	Non-scattering potentials within the Born approximation	266
	6.9.2	Incident plane waves	267
	6.9.3	The relationship between the two Born non-scattering potentials	269
	6.9.4	Almost-invisible weak scatterers	269
	6.9.5	Essentially non-scattering potentials	270
6.10	The Ry	tov approximation	271
	6.10.1	The Ricatti equation for the complex phase of the field	272
	6.10.2	The Liouville–Neumann expansion for the phase	273
	6.10.3	The Rytov approximation	274
	6.10.4	The short-wavelength limit	274
	6.10.5	The Rytov transformation	275
	6.10.6	A comparison of the Born and Rytov approximations	276
	6.10.7	The hybrid approximation	278
6.11		t spherical waves and slant stacking	280
	6.11.1	Slant stacking from arbitrary surfaces	282
	6.11.2	Slant-stack computation of the scattering amplitude	282
	er reading	g	283
Probl	ems		283
C.,£		an and diffraction	305
5urta 7.1		ng and diffraction ation of the scattering problem for non-penetrable scatterers	285 286
7.1	LOHHIIII	ation of the scattering problem for holl-beneficiable scatterers	Z00

		7.1.1	The scattering amplitude	287
		7.1.2	Liouville–Neumann expansion	288
	7.2	Scatter	ing from simple shapes	289
		7.2.1	Scattering from an infinite Dirichlet or Neumann plane	289
		7.2.2	Scattering from a sphere	291
		7.2.3	Scattering of a plane wave from a cylinder	293
	7.3	The ph	ysical-optics approximation	295
		7.3.1	Plane-wave incidence	298
		7.3.2	Simulation	299
	7.4	The Bo	ojarski transformation and linearized inverse surface scattering	300
		7.4.1	The generalized Bojarski transformation	304
		7.4.2	Inverse scattering within the PO approximation	305
	7.5	Kirchh	off diffraction theory	306
		7.5.1	The Rayleigh-Sommerfeld alternative to the Kirchhoff	
			diffraction formula	308
		7.5.2	More general diffraction problems	309
		7.5.3	Algorithmic implementation of the diffraction formulas	310
	7.6	Inverse	e diffraction	314
		7.6.1	Inverse diffraction using back propagation	315
		7.6.2	The SVD formulation of the inverse diffraction problem	316
		7.6.3	The full data case	316
		7.6.4	The Slepian–Pollak theory	319
	7.7	Detern	nining the shape of a surface scatterer	320
		7.7.1	Surface reconstruction via back propagation	321
		7.7.2	The SVD approach to surface reconstruction	325
	Furth	er readin	ng	331
	Proble	ems		332
8	Classic	al invers	e scattering and diffraction tomography	333
	8.1	Born i	nverse scattering from far-field data	335
		8.1.1	Born inversion from ideal data	336
		8.1.2	The filtered back-propagation algorithm	339
		8.1.3	Inverse scattering identity	340
		8.1.4	The FBP algorithm in two space dimensions	342
	8.2	Born i	nversion from limited and non-perfect data	344
		8.2.1	Non-perfect data	345
		8.2.2	Limited-data case I	346
		8.2.3	Limited-data case II	346
	8.3	Non-u	niqueness and non-scattering scatterers	347
		8.3.1	Non-scattering potentials within the Born approximation	348
	8.4	Hilber	rt-space formulation of Born inverse scattering	349
		8.4.1	Adjoint and composite operators	350
		8.4.2	Singular value decomposition	351
		8.4.3	Solution to the inverse scattering problem	353

	8.5		nversion using non-plane-wave probes and arbitrary measure-	a
		ment si		354
		8.5.1	Data collected on arbitrary surfaces	354
	0.6	8.5.2	Incident spherical waves	357
	8.6		re algorithms	357
		8.6.1	Limited-data problems	358
	0.=	8.6.2	Incorporation of constraints	359
	8.7	-	graphic formulation of inverse scattering	360
		8.7.1	The Rytov approximation	361
		8.7.2	The short-wavelength limit of DT	361
		8.7.3	Computed tomography	362
		8.7.4	The projection-slice theorem	364
		8.7.5	The filtered back-projection algorithm	364
		8.7.6	Computed tomography of circularly symmetric objects	366
	8.8		ction tomography	367
		8.8.1	Hybrid formulation	367
		8.8.2	Reduction to a set of 2D inverse scattering problems	368
	8.9		ction tomography in two space dimensions	370
		8.9.1	The generalized projection-slice theorem	371
		8.9.2	The filtered back-propagation algorithm	373
		8.9.3	Diffraction tomography of circularly symmetric objects	375
	8.10		ations of DT with ideal Rytov data	378
	8.11		dimensional diffraction tomography	381
		er readin	ng	384
	Proble	ems		386
9	Waves		nogeneous media	387
	9.1	_	round-medium Green functions	388
		9.1.1	The reciprocity condition for the Green functions	389
		9.1.2	Plane-wave scattering states	390
	9.2	The ra	diation problem in non-uniform backgrounds	393
		9.2.1	The Green-function solution to the radiation problem	393
		9.2.2	The Kirchhoff-Helmholtz representation of the radiated field	394
		9.2.3	The Porter–Bojarski integral equation	395
		9.2.4	The radiation pattern of the field	396
	9.3	Genera	alized plane-wave expansions	397
		9.3.1	Generalized plane-wave expansions to the homogeneous	
			Helmholtz equation in a non-uniform medium	397
		9.3.2	Generalized angular-spectrum expansions	400
		9.3.3	Angular-spectrum expansion of the radiated field in non-	
			uniform media	401
	9.4	Non-ra	adiating sources in non-uniform media	402
		9.4.1	Non-radiating sources and the radiation pattern	403
	9.5	The in	verse source problem	404

		9.5.1	General formulation	405
		9.5.2	Singular value decomposition	406
		9.5.3	The least-squares pseudo-inverse solution of the ISP	407
	9.6	Solution	n of the ISP for spherically symmetric backgrounds	408
		9.6.1	Solution of the ISP for a piecewise-constant spherically	
			symmetric background	411
		9.6.2	Super-resolution	411
	9.7	Scatteri	ng in a non-uniform background medium	413
	9.8	The dist	torted-wave Born approximation	415
		9.8.1	The DWBA for a pair of concentric homogeneous cylinders	416
	9.9	Foldy-I	Lax theory	422
	9.10	Inverse	scattering within the DWBA	423
		9.10.1	The far-field limited-view ISCP	424
		9.10.2	Back-propagation imaging	425
		9.10.3	The limited-view problem in a homogeneous background	426
	9.11	The ISC	CP using data generated and acquired by sets of antennas	429
	Furthe	r reading		432
	Proble	ems		432
10	Time-re	eversal im	aging for systems of discrete scatterers	434
	10.1	Experin	nental time-reversal imaging	435
		10.1.1	Experimental time-reversal imaging in non-uniform media	436
	10.2	Time-re	eversal imaging using a finite set of antennas	438
		10.2.1	Experimental time-reversal imaging	438
		10.2.2	Eigenvectors of the time-reversal matrix	440
		10.2.3	Focusing with the eigenvectors of the time-reversal matrix	443
	10.3	Comput	tational time-reversal imaging	444
		10.3.1	Singular value decomposition of the multistatic data matrix	446
		10.3.2	DORT	447
		10.3.3	Time-reversal MUSIC	452
		10.3.4	Filtered DORT and multiple-frequency algorithms	453
	10.4	The inv	erse scattering problem	456
	Furthe	r reading		457
	Proble	ems		457
11	The ele	ctromagr	netic field	459
	11.1	Maxwe	ll equations	459
		11.1.1	Maxwell equations for a homogeneous isotropic medium	460
		11.1.2	Maxwell equations in the spatial frequency domain	461
	11.2	The He	lmholtz theorem	461
	11.3	The EM	I radiation problem	464
		11.3.1	The dyadic Green function	464
		11.3.2	The radiation patterns	466
		11.3.3	The Kirchhoff–Helmholtz representation of the radiated field	467

11.4	Angular-spectrum expansions of the radiated field	468
	11.4.1 The angle-variable form of the angular-spectrum expansion	
	of the EM field	469
	11.4.2 Back propagation from the radiation patterns	470
	11.4.3 The Cartesian-variable form of the angular-spectrum	
	expansion of the EM field	471
	11.4.4 Forward and back propagation from planar boundary-value data	472
11.5	The Whittaker representation of the radiated fields	473
	11.5.1 Boundary-value problems and field back propagation using	
	the Whittaker representation	476
11.6	Debye representation and multipole expansions of radiated fields	479
	11.6.1 Multipole expansion of the radiated field	482
11.7	Vector spherical-harmonic expansion of the radiation pattern	484
11.8	The EM inverse source problem	485
	11.8.1 The EM ISP for sources supported in spherical regions	485
11.9	Electromagnetic scattering theory	488
	11.9.1 The Lippmann–Schwinger equations	489
	11.9.2 Electromagnetic scattering amplitudes	490
	11.9.3 angular-spectrum expansions	490
11.10	The Born approximation	491
	11.10.1 Born scattering amplitudes	492
	11.10.2 Born inverse scattering	493
Furthe	r reading	494
Proble	ems	494
Appendix A	A Proof of the scattering amplitude theorems	496
A.1	Proof of the reciprocity theorem	496
A.2	Proof of the translation theorem	497
A.3	Proof of the optical theorem	497
Appendix I	B Derivation of the generalized Weyl expansion	501
B.1	Off-shell Green functions and scattering wave states	501
B.2	Derivation of the generalized Weyl expansion	502
Reference	es	505
Index		515