

Contents

Preface	ix
1 Polynomials	1
1.1 Definitions	1
1.2 Multiplication and Degree	4
1.3 Factorization and Roots	8
1.4 Bounding the Number of Roots	10
1.5 Real Numbers and the Intermediate Value Theorem	12
1.6 Graphs	16
2 Quadratic Polynomials	21
2.1 Sums and Products	22
2.2 Completing the Square	24
2.3 Changing Variables	28
2.4 A Discriminant	29
2.5 History	33
3 Cubic Polynomials	47
3.1 Reduced Cubics	47
3.2 Cardano's Formula	50
3.3 Graphs	58
3.4 A Discriminant	61
3.5 History	66
4 Complex Numbers	73
4.1 Complex Numbers	73
4.2 Quadratic Polynomials and the Discriminant	77
4.3 Square and Cube Roots	81

4.4	The Complex Plane	84
4.5	A Geometric Interpretation of Multiplication	88
4.6	Euler's and de Moivre's Formulas	92
4.7	Roots of Unity	98
4.8	Converting Root Extraction to Division	101
4.9	History	103
5	Cubic Polynomials, II	109
5.1	Cardano's formula	109
5.2	The Resolvent	113
5.3	The Discriminant	115
5.4	Cardano's Formula Refined	120
5.5	The Irreducible Case	124
5.6	Viète's Formula	125
5.7	The Signs of the Real Roots	130
5.8	History	133
6	Quartic Polynomials	143
6.1	Reduced Quartics	143
6.2	Ferrari's Method	146
6.3	Descartes' Method	149
6.4	Euler's Formula	154
6.5	The Discriminant	157
6.6	The Nature of the Roots	162
6.7	Cubic and Quartic Reprise	167
6.8	History	169
7	Higher-Degree Polynomials	179
7.1	Quintic Polynomials	179
7.2	The Fundamental Theorem of Algebra	185
7.3	Polynomial Factorization	191
7.4	Symmetric Polynomials	200
7.5	A Proof of the Fundamental Theorem	211
	References	217
	Index	223
	About the Author	227