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- C.2 The Cartesian Plane
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**Appendix D: Rotation and the General Second-Degree Equation (Web)\*****Appendix E: Complex Numbers (Web)\*****Appendix F: Business and Economic Applications (Web)\***

Answers to All Odd-Numbered Exercises and Tests A7

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