

Contents

I	The Basics	1
1	The Equations of Fluids	3
1.1	Symbols	3
1.2	The Momentum Equation	4
1.3	Lagrangian and Eulerian Viewpoints	7
1.4	Incompressibility	11
1.5	Dropping Viscosity	13
1.6	Boundary Conditions	13
2	Overview of Numerical Simulation	17
2.1	Splitting	17
2.2	Splitting the Fluid Equations	19
2.3	Time Steps	20
2.4	Grids	21
2.5	Dynamic Sparse Grids	25
2.6	Two Dimensional Simulations	27
3	Advection Algorithms	29
3.1	Semi-Lagrangian Advection	29
3.2	Boundary Conditions	33
3.3	Time Step Size	34
3.4	Diffusion	37
3.5	Reducing Numerical Diffusion	39
4	Level Set Geometry	43
4.1	Signed Distance	44
4.2	Discretizing Signed Distance Functions	47
4.3	Computing Signed Distance	49
4.4	Recomputing Signed Distance	54
4.5	Operations on Level Sets	55
4.6	Contouring	59
4.7	Limitations of Level Sets	64
4.8	Extrapolating Data	64

5	Making Fluids Incompressible	67
5.1	The Discrete Pressure Gradient	68
5.2	The Discrete Divergence	72
5.3	The Pressure Equations	74
5.4	Projection	90
5.5	More Accurate Curved Boundaries	91
5.6	The Compatibility Condition	96
6	Smoke	99
6.1	Temperature and Smoke Concentration	99
6.2	Buoyancy	102
6.3	Variable Density Solves	103
6.4	Divergence Control	105
7	Particle Methods	107
7.1	Advection Troubles on Grids	107
7.2	Particle Advection	109
7.3	Transferring Particles to the Grid	111
7.4	Particle Seeding	114
7.5	Diffusion	115
7.6	Particle-in-Cell Methods	116
II	More Types of Fluids	121
8	Water	123
8.1	Marker Particles and Voxels	123
8.2	More Accurate Pressure Solves	127
8.3	Topology Change and Wall Separation	129
8.4	Volume Control	130
8.5	Surface Tension	131
9	Fire	133
9.1	Thin Flames	134
9.2	Volumetric Combustion	137
10	Viscous Fluids	139
10.1	Stress	139
10.2	Applying Stress	141
10.3	Strain Rate and Newtonian Fluids	142
10.4	Boundary Conditions	147
10.5	Implementation	148

III	More Algorithms	161
11	Turbulence	163
11.1	Vorticity	163
11.2	Vorticity Confinement	167
11.3	Procedural Turbulence	168
11.4	Simulating Sub-Grid Turbulence	172
12	Shallow Water	175
12.1	Deriving the Shallow Water Equations	176
12.2	The Wave Equation	180
12.3	Discretization	182
13	Ocean Modeling	185
13.1	Potential Flow	185
13.2	Simplifying Potential Flow for the Ocean	188
13.3	Evaluating the Height Field Solution	193
13.4	Unsimplifying the Model	195
13.5	Wave Parameters	198
13.6	Eliminating Periodicity	199
14	Vortex Methods	201
14.1	Velocity from Vorticity	202
14.2	Biot-Savart and Streamfunctions	206
14.3	Vortex Particles	207
15	Coupling Fluids and Solids	217
15.1	One-Way Coupling	217
15.2	Weak Coupling	219
15.3	The Immersed Boundary Method	222
15.4	General Sparse Matrices	223
15.5	Strong Coupling	225
A	Background	231
A.1	Vector Calculus	231
A.2	Numerical Methods	239
B	Derivations	243
B.1	The Incompressible Euler Equations	243
	Bibliography	247
	Index	257