

CONTENTS

PREFACE	xv
ACKNOWLEDGMENTS	xix
ABBREVIATIONS	xxi
1 INTRODUCTION	1
2 LEAST-SQUARES ADJUSTMENTS	11
2.1 Elementary Considerations / 12	
2.1.1 Statistical Nature of Surveying Measurements / 12	
2.1.2 Observational Errors / 13	
2.1.3 Accuracy and Precision / 13	
2.2 Stochastic and Mathematical Models / 14	
2.3 Mixed Model / 17	
2.3.1 <i>Linearization</i> / 18	
2.3.2 Minimization and Solution / 19	
2.3.3 Cofactor Matrices / 20	
2.3.4 A Posteriori Variance of Unit Weight / 21	
2.3.5 Iterations / 22	
2.4 Sequential Mixed Model / 23	
2.5 Model Specifications / 29	
2.5.1 Observation Equation Model / 29	
2.5.2 Condition Equation Model / 30	

2.5.3	Mixed Model with Observation Equations /	30
2.5.4	Sequential Observation Equation Model /	32
2.5.5	Observation Equation Model with Observed Parameters /	32
2.5.6	Mixed Model with Conditions /	34
2.5.7	Observation Equation Model with Conditions /	35
2.6	Minimal and Inner Constraints /	37
2.7	Statistics in Least-Squares Adjustment /	42
2.7.1	Fundamental Test /	42
2.7.2	Testing Sequential Least Squares /	48
2.7.3	General Linear Hypothesis /	49
2.7.4	Ellipses as Confidence Regions /	52
2.7.5	Properties of Standard Ellipses /	56
2.7.6	Other Measures of Precision /	60
2.8	Reliability /	62
2.8.1	Redundancy Numbers /	62
2.8.2	Controlling Type-II Error for a Single Blunder /	64
2.8.3	Internal Reliability /	67
2.8.4	Absorption /	67
2.8.5	External Reliability /	68
2.8.6	Correlated Cases /	69
2.9	Blunder Detection /	70
2.9.1	Tau Test /	71
2.9.2	Data Snooping /	71
2.9.3	Changing Weights of Observations /	72
2.10	Examples /	72
2.11	Kalman Filtering /	77

3 RECURSIVE LEAST SQUARES

81

3.1	Static Parameter /	82
3.2	Static Parameters and Arbitrary Time-Varying Variables /	87
3.3	Dynamic Constraints /	96
3.4	Static Parameters and Dynamic Constraints /	112
3.5	Static Parameter, Parameters Subject to Dynamic Constraints, and Arbitrary Time-Varying Parameters /	125

4 GEODESY

129

4.1	International Terrestrial Reference Frame /	131
4.1.1	Polar Motion /	132
4.1.2	Tectonic Plate Motion /	133
4.1.3	Solid Earth Tides /	135

- 4.1.4 Ocean Loading / 135
- 4.1.5 Relating of Nearly Aligned Frames / 136
- 4.1.6 ITRF and NAD83 / 138
- 4.2 International Celestial Reference System / 141
 - 4.2.1 Transforming Terrestrial and Celestial Frames / 143
 - 4.2.2 Time Systems / 149
- 4.3 Datum / 151
 - 4.3.1 Geoid / 152
 - 4.3.2 Ellipsoid of Rotation / 157
 - 4.3.3 Geoid Undulations and Deflections of the Vertical / 158
 - 4.3.4 Reductions to the Ellipsoid / 162
- 4.4 3D Geodetic Model / 166
 - 4.4.1 Partial Derivatives / 169
 - 4.4.2 Reparameterization / 170
 - 4.4.3 Implementation Considerations / 171
 - 4.4.4 GPS Vector Networks / 174
 - 4.4.5 Transforming Terrestrial and Vector Networks / 176
 - 4.4.6 GPS Network Examples / 178
 - 4.4.6.1 Montgomery County Geodetic Network / 178
 - 4.4.6.2 SLC Engineering Survey / 182
 - 4.4.6.3 Orange County Densification / 183
- 4.5 Ellipsoidal Model / 190
 - 4.5.1 Reduction of Observations / 191
 - 4.5.1.1 Angular Reduction to Geodesic / 192
 - 4.5.1.2 Distance Reduction to Geodesic / 193
 - 4.5.2 Direct and Inverse Solutions on the Ellipsoid / 195
 - 4.5.3 Network Adjustment on the Ellipsoid / 196
- 4.6 Conformal Mapping Model / 197
 - 4.6.1 Reduction of Observations / 198
 - 4.6.2 Angular Excess / 200
 - 4.6.3 Direct and Inverse Solutions on the Map / 201
 - 4.6.4 Network Adjustment on the Map / 201
 - 4.6.5 Similarity Revisited / 203
- 4.7 Summary / 204

5 SATELLITE SYSTEMS

207

- 5.1 Motion of Satellites / 207
 - 5.1.1 Kepler Elements / 208
 - 5.1.2 Normal Orbital Theory / 210
 - 5.1.3 Satellite Visibility and Topocentric Motion / 219

5.1.4	Perturbed Satellite Motion /	219
5.1.4.1	Gravitational Field of the Earth /	220
5.1.4.2	Acceleration due to the Sun and the Moon /	222
5.1.4.3	Solar Radiation Pressure /	222
5.1.4.4	Eclipse Transits and Yaw Maneuvers /	223
5.2	Global Positioning System /	225
5.2.1	General Description /	226
5.2.2	Satellite Transmissions at 2014 /	228
5.2.2.1	Signal Structure /	229
5.2.2.2	Navigation Message /	237
5.2.3	GPS Modernization Comprising Block IIM, Block IIF, and Block III /	239
5.2.3.1	Introducing Binary Offset Carrier (BOC) Modulation /	241
5.2.3.2	Civil L2C Codes /	243
5.2.3.3	Civil L5 Code /	243
5.2.3.4	M-Code /	244
5.2.3.5	Civil L1C Code /	244
5.3	GLONASS /	245
5.4	Galileo /	248
5.5	QZSS /	250
5.6	Beidou /	252
5.7	IRNSS /	254
5.8	SBAS: WAAS, EGNOS, GAGAN, MSAS, and SDCM /	254

6 GNSS POSITIONING APPROACHES

257

6.1	Observables /	258
6.1.1	Undifferenced Functions /	261
6.1.1.1	Pseudoranges /	261
6.1.1.2	Carrier Phases /	263
6.1.1.3	Range plus Ionosphere /	266
6.1.1.4	Ionospheric-Free Functions /	266
6.1.1.5	Ionospheric Functions /	267
6.1.1.6	Multipath Functions /	267
6.1.1.7	Ambiguity-Corrected Functions /	268
6.1.1.8	Triple-Frequency Subscript Notation /	269
6.1.2	Single Differences /	271
6.1.2.1	Across-Receiver Functions /	271
6.1.2.2	Across-Satellite Functions /	272
6.1.2.3	Across-Time Functions /	272

- 6.1.3 Double Differences / 273
- 6.1.4 Triple Differences / 275
- 6.2 Operational Details / 275
 - 6.2.1 Computing the Topocentric Range / 275
 - 6.2.2 Satellite Timing Considerations / 276
 - 6.2.2.1 Satellite Clock Correction and Timing Group Delay / 278
 - 6.2.2.2 Intersignal Correction / 279
 - 6.2.3 Cycle Slips / 282
 - 6.2.4 Phase Windup Correction / 283
 - 6.2.5 Multipath / 286
 - 6.2.6 Phase Center Offset and Variation / 292
 - 6.2.6.1 Satellite Phase Center Offset / 292
 - 6.2.6.2 User Antenna Calibration / 293
 - 6.2.7 GNSS Services / 295
 - 6.2.7.1 IGS / 295
 - 6.2.7.2 Online Computing / 298
- 6.3 Navigation Solution / 299
 - 6.3.1 Linearized Solution / 299
 - 6.3.2 DOPs and Singularities / 301
 - 6.3.3 Nonlinear Closed Solution / 303
- 6.4 Relative Positioning / 304
 - 6.4.1 Nonlinear Double-Difference Pseudorange Solution / 305
 - 6.4.2 Linearized Double- and Triple-Differenced Solutions / 306
 - 6.4.3 Aspects of Relative Positioning / 310
 - 6.4.3.1 Singularities / 310
 - 6.4.3.2 Impact of a Priori Position Error / 311
 - 6.4.3.3 Independent Baselines / 312
 - 6.4.3.4 Antenna Swap Technique / 314
 - 6.4.4 Equivalent Undifferenced Formulation / 315
 - 6.4.5 Ambiguity Function / 316
 - 6.4.6 GLONASS Carrier Phase / 319
- 6.5 Ambiguity Fixing / 324
 - 6.5.1 The Constraint Solution / 324
 - 6.5.2 LAMBDA / 327
 - 6.5.3 Discernibility / 334
 - 6.5.4 Lattice Reduction and Integer Least Squares / 337
 - 6.5.4.1 Branch-and-Bound Approach / 338
 - 6.5.4.2 Finke-Pohst Algorithm / 349
 - 6.5.4.3 Lattice Reduction Algorithms / 351

6.5.4.4	Other Searching Strategies /	354
6.5.4.5	Connection Between LAMBDA and LLL Methods /	356
6.6	Network-Supported Positioning /	357
6.6.1	PPP /	357
6.6.2	CORS /	363
6.6.2.1	Differential Phase and Pseudorange Corrections /	363
6.6.2.2	RTK /	365
6.6.3	PPP-RTK /	367
6.6.3.1	Single-Frequency Solution /	367
6.6.3.2	Dual-Frequency Solutions /	372
6.6.3.3	Across-Satellite Differencing /	379
6.7	Triple-Frequency Solutions /	382
6.7.1	Single-Step Position Solution /	382
6.7.2	Geometry-Free TCAR /	386
6.7.2.1	Resolving EWL Ambiguity /	389
6.7.2.2	Resolving the WL Ambiguity /	391
6.7.2.3	Resolving the NL Ambiguity /	393
6.7.3	Geometry-Based TCAR /	395
6.7.4	Integrated TCAR /	396
6.7.5	Positioning with Resolved Wide Lanes /	397
6.8	Summary /	398

7 REAL-TIME KINEMATICS RELATIVE POSITIONING

401

7.1	Multisystem Considerations /	402
7.2	Undifferenced and Across-Receiver Difference Observations /	403
7.3	Linearization and Hardware Bias Parameterization /	408
7.4	RTK Algorithm for Static and Short Baselines /	418
7.4.1	Illustrative Example /	422
7.5	RTK Algorithm for Kinematic Rovers and Short Baselines /	429
7.5.1	Illustrative Example /	431
7.6	RTK Algorithm with Dynamic Model and Short Baselines /	435
7.6.1	Illustrative Example /	437
7.7	RTK Algorithm with Dynamic Model and Long Baselines /	441
7.7.1	Illustrative Example /	442
7.8	RTK Algorithms with Changing Number of Signals /	445
7.9	Cycle Slip Detection and Isolation /	450
7.9.1	Solutions Based on Signal Redundancy /	455
7.10	Across-Receiver Ambiguity Fixing /	466
7.10.1	Illustrative Example /	470
7.11	Software Implementation /	473

8 TROPOSPHERE AND IONOSPHERE 475

- 8.1 Overview / 476
- 8.2 Tropospheric Refraction and Delay / 479
 - 8.2.1 Zenith Delay Functions / 482
 - 8.2.2 Mapping Functions / 482
 - 8.2.3 Precipitable Water Vapor / 485
- 8.3 Troposphere Absorption / 487
 - 8.3.1 The Radiative Transfer Equation / 487
 - 8.3.2 Absorption Line Profiles / 490
 - 8.3.3 General Statistical Retrieval / 492
 - 8.3.4 Calibration of WVR / 494
- 8.4 Ionospheric Refraction / 496
 - 8.4.1 Index of Ionospheric Refraction / 499
 - 8.4.2 Ionospheric Function and Cycle Slips / 504
 - 8.4.3 Single-Layer Ionospheric Mapping Function / 505
 - 8.4.4 VTEC from Ground Observations / 507
 - 8.4.5 Global Ionospheric Maps / 509
 - 8.4.5.1 IGS GIMs / 509
 - 8.4.5.2 International Reference Ionosphere / 509
 - 8.4.5.3 GPS Broadcast Ionospheric Model / 510
 - 8.4.5.4 NeQuick Model / 510
 - 8.4.5.5 Transmission to the User / 511

9 GNSS RECEIVER ANTENNAS 513

- 9.1 Elements of Electromagnetic Fields and Electromagnetic Waves / 515
 - 9.1.1 Electromagnetic Field / 515
 - 9.1.2 Plane Electromagnetic Wave / 518
 - 9.1.3 Complex Notations and Plane Wave in Lossy Media / 525
 - 9.1.4 Radiation and Spherical Waves / 530
 - 9.1.5 Receiving Mode / 536
 - 9.1.6 Polarization of Electromagnetic Waves / 537
 - 9.1.7 The dB Scale / 544
- 9.2 Antenna Pattern and Gain / 546
 - 9.2.1 Receiving GNSS Antenna Pattern and Reference Station and Rover Antennas / 546
 - 9.2.2 Directivity / 553
 - 9.2.3 Polarization Properties of the Receiving GNSS Antenna / 558
 - 9.2.4 Antenna Gain / 562
 - 9.2.5 Antenna Effective Area / 564

- 9.3 Phase Center / 565
 - 9.3.1 Antenna Phase Pattern / 566
 - 9.3.2 Phase Center Offset and Variations / 568
 - 9.3.3 Antenna Calibrations / 575
 - 9.3.4 Group Delay Pattern / 577
- 9.4 Diffraction and Multipath / 578
 - 9.4.1 Diffraction Phenomena / 578
 - 9.4.2 General Characterization of Carrier Phase Multipath / 585
 - 9.4.3 Specular Reflections / 587
 - 9.4.4 Antenna Down-Up Ratio / 593
 - 9.4.5 PCV and PCO Errors Due to Ground Multipath / 597
- 9.5 Transmission Lines / 600
 - 9.5.1 Transmission Line Basics / 600
 - 9.5.2 Antenna Frequency Response / 606
 - 9.5.3 Cable Losses / 608
- 9.6 Signal-to-Noise Ratio / 609
 - 9.6.1 Noise Temperature / 609
 - 9.6.2 Characterization of Noise Sources / 611
 - 9.6.3 Signal and Noise Propagation through a Chain of Circuits / 615
 - 9.6.4 SNR of the GNSS Receiving System / 619
- 9.7 Antenna Types / 620
 - 9.7.1 Patch Antennas / 620
 - 9.7.2 Other Types of Antennas / 629
 - 9.7.3 Flat Metal Ground Planes / 629
 - 9.7.4 Impedance Ground Planes / 634
 - 9.7.5 Vertical Choke Rings and Compact Rover Antenna / 642
 - 9.7.6 Semitransparent Ground Planes / 644
 - 9.7.7 Array Antennas / 645
 - 9.7.8 Antenna Manufacturing Issues / 650

APPENDIXES

A GENERAL BACKGROUND

- A.1 Spherical Trigonometry / 653
- A.2 Rotation Matrices / 657
- A.3 Linear Algebra / 657
 - A.3.1 Determinants and Matrix Inverse / 658
 - A.3.2 Eigenvalues and Eigenvectors / 659

A.3.3	Eigenvalue Decomposition /	660
A.3.4	Quadratic Forms /	661
A.3.5	Matrix Partitioning /	664
A.3.6	Cholesky Decomposition /	666
A.3.7	Partial Minimization of Quadratic Functions /	669
A.3.8	QR Decomposition /	673
A.3.9	Rank One Update of Cholesky Decomposition /	676
A.4	Linearization /	681
A.5	Statistics /	683
A.5.1	One-Dimensional Distributions /	683
A.5.2	Distribution of Simple Functions /	688
A.5.3	Hypothesis Tests /	689
A.5.4	Multivariate Distributions /	691
A.5.5	Variance-Covariance Propagation /	693
A.5.6	Multivariate Normal Distribution /	695

B THE ELLIPSOID

697

B.1	Geodetic Latitude, Longitude, and Height /	698
B.2	Computation of the Ellipsoidal Surface /	703
B.2.1	Fundamental Coefficients /	703
B.2.2	Gauss Curvature /	705
B.2.3	Elliptic Arc /	706
B.2.4	Angle /	706
B.2.5	Isometric Latitude /	707
B.2.6	Differential Equation of the Geodesic /	708
B.2.7	The Gauss Midlatitude Solution /	711
B.2.8	Angular Excess /	713
B.2.9	Transformation in a Small Region /	713

C CONFORMAL MAPPING

715

C.1	Conformal Mapping of Planes /	716
C.2	Conformal Mapping of General Surfaces /	719
C.3	Isometric Plane /	721
C.4	Popular Conformal Mappings /	722
C.4.1	Equatorial Mercator /	723
C.4.2	Transverse Mercator /	724
C.4.3	Lambert Conformal /	726
C.4.4	SPC and UTM /	738

D	VECTOR CALCULUS AND DELTA FUNCTION	741
E	ELECTROMAGNETIC FIELD GENERATED BY ARBITRARY SOURCES, MAGNETIC CURRENTS, BOUNDARY CONDITIONS, AND IMAGES	747
F	DIFFRACTION OVER HALF-PLANE	755
G	SINGLE CAVITY MODE APPROXIMATION WITH PATCH ANTENNA ANALYSIS	759
H	PATCH ANTENNAS WITH ARTIFICIAL DIELECTRIC SUBSTRATES	763
I	CONVEX PATCH ARRAY GEODETIC ANTENNA	769
	REFERENCES	773
	AUTHOR INDEX	793
	SUBJECT INDEX	801