Contents

Preface ix

Acknowledgments xiii

1 Introductory Concepts 1

1.1 Introduction . . . 1
1.1.1 What Is a System? . . . 1
1.2 Particles, Rigid Bodies, and Deformable Solids . . . 2
1.3 Degrees of Freedom . . . 3
1.3.1 Constraints . . . 5
1.3.2 Generalized Coordinates . . . 7
1.4 Types of Forces and Motions . . . 9
1.4.1 Vibratory Motion . . . 10
1.5 Systems of Units . . . 10
1.6 Linearization . . . 12
1.7 Differential Equations and the Principle of Superposition . . . 15
1.8 Dimensional Analysis and Nondimensionalization . . . 17
1.9 Numerical Integration . . . 18
1.10 What Is a Vehicle? . . . 20
1.11 Cause-and-Effect Principle . . . 20
1.12 Bibliography . . . 21
1.13 Problems . . . 21

2 Kinematics Fundamentals **25**

2.1 Introduction . . . 25
2.2 Position, Velocity, and Acceleration . . . 25
2.3 Reference Frames: Single Rotation in a Plane . . . 26
2.4 Column Vector Representation . . . 29
2.5 Commonly Used Coordinate Systems . . . 30
2.5.1 Rectilinear Coordinates . . . 31
2.5.2 Normal-Tangential Coordinates . . . 33
2.5.3 Cylindrical Coordinates . . . 38
2.5.4 Spherical Coordinates . . . 42
2.6 Moving Reference Frames . . . 46
2.7 Selection of Rotation Parameters . . . 48
2.7.1 Transformation by Three Rotation Angles . . . 49
2.7.2 Resolving a Rotated Vector . . . 51
2.7.3 Single Rotation about a Specified Axis . . . 52
2.7.4 Finite Rotations Do Not Commute . . . 54
2.8 Rate of Change of a Vector, Angular Velocity . . . 58
2.8.1 Angular Velocity for Plane Motion . . . 58
2.8.2 Angular Velocity for Three-Dimensional Motion . . . 59

2.8.3 Other Definitions of Angular Velocity . . . 62
2.8.4 Additive Properties of Angular Velocity . . . 62
2.9 Angular Acceleration and Second Derivatives . . . 64
2.9.1 Angular Acceleration . . . 64
2.9.2 Second Derivatives . . . 67
2.10 Relative Motion . . . 69
2.11 Instantaneous Center of Zero Velocity . . . 73
2.12 Bibliography . . . 74
2.13 Problems . . . 75

3 Kinematics Applications **83**
3.1 Introduction . . . 83
3.2 Motion with Respect to the Rotating Earth . . . 83
3.3 Contact . . . 85
3.4 Rolling . . . 86
3.4.1 General Formulation . . . 87
3.4.2 Rolling Constraints, Wheel on an Axle . . . 88
3.5 Bicycle Model of a Car . . . 92
3.5.1 Where Is the Instant Center of a Car? . . . 95
3.6 Kinematic Differential Equations . . . 98
3.7 Topspin and Backspin . . . 99
3.8 Mechanisms . . . 100
3.8.1 Links and Joints . . . 101
3.8.2 Degrees of Freedom: Gruebler's Equation . . . 102
3.8.3 Four-Bar Linkage and Slider-Crank Mechanism . . . 104
3.9 Instant Center Analysis for Linkages . . . 105
3.9.1 Locating Instant Centers in Linkages . . . 105
3.9.2 Velocity Analysis Using Instant Centers . . . 108
3.10 Bibliography . . . 110
3.11 Problems . . . 110

4 Kinetics Fundamentals **115**
4.1 Introduction . . . 115
4.2 Rigid Body Geometry . . . 115
4.2.1 Center of Mass . . . 115
4.2.2 Mass Moment of Inertia . . . 118
4.3 Linear Momentum and Angular Momentum . . . 120
4.4 Resultant Force and Moment . . . 124
4.5 Laws of Motion . . . 125
4.5.1 First Law . . . 126
4.5.2 Second Law . . . 126
4.5.3 Third Law . . . 128
4.5.4 Inertia Forces and Inertia Moments . . . 129
4.6 Forces and Moments Acting on Bodies . . . 131
4.7 Force of Gravity . . . 132
4.8 Contact and Reaction Forces . . . 133
4.9 Dry Friction Forces . . . 134
4.10 Aerodynamic Forces . . . 138
4.10.1 Lift Force and Drag Force . . . 140
4.10.2 Aerodynamic Coefficients . . . 141
4.10.3 Flow Separation . . . 143

4.10.4 Drag Approximation for Very Low Reynolds Numbers . . . 145
4.11 Spring Forces . . . 147
4.11.1 Modeling of Springs . . . 148
4.11.2 Equivalent Spring Constants . . . 149
4.11.3 Stiffness Generating Components . . . 151
4.12 Dampers . . . 155
4.13 Bibliography . . . 156
4.14 Problems . . . 156

5 Kinetics Applications **165**
5.1 Introduction . . . 165
5.2 Rolling . . . 165
5.3 Mechanical Trail . . . 169
5.4 Impulse and Momentum . . . 172
5.4.1 Impulsive Forces . . . 172
5.4.2 Idealized Model of an Impulsive Force . . . 174
5.5 Work, Energy, and Power . . . 175
5.5.1 Kinetic Energy and Power . . . 175
5.5.2 Potential Energy . . . 178
5.5.3 Gravitational Potential Energy . . . 179
5.5.4 Potential Energy of Springs . . . 181
5.5.5 Work-Energy Relations . . . 182
5.5.6 Forces That Do No Work . . . 182
5.5.7 Hysteresis and Energy Loss . . . 183
5.6 Equations of Motion . . . 184
5.6.1 Deriving the Equation of Motion of Conservative One-Degree-of-Freedom Systems Using Energy . . . 188
5.7 Solution of the Equations of Motion . . . 189
5.8 Linearization, Equilibrium, and Stability . . . 190
5.8.1 Calculating the Equilibrium Position(s) . . . 191
5.8.2 Motion in the Vicinity of Equilibrium . . . 191
5.8.3 Nature of the Response of the Linearized Equations . . . 193
5.8.4 Equations of Motion of Linear Systems about Equilibrium . . . 193
5.9 Motion in the Vicinity of the Earth . . . 200
5.10 Collisions . . . 203
5.10.1 Collisions of Particles . . . 203
5.11 Impact of Rigid Bodies: Simple Solution . . . 208
5.12 A More Accurate Model of Rigid Body Impact . . . 214
5.12.1 Compression Stage . . . 216
5.12.2 Restitution Stage . . . 218
5.13 Bibliography . . . 220
5.14 Problems . . . 221

6 Response of Dynamical Systems **231**
6.1 Introduction . . . 231
6.2 The Unit Impulse and Unit Step Functions . . . 231
6.2.1 The Unit Impulse Function . . . 231
6.2.2 The Unit Step Function . . . 232
6.3 Homogeneous Plus Particular Solution Approach . . . 233
6.4 Laplace Transform Solution . . . 235
6.4.1 General Formulation . . . 235

6.4.2 Solving Differential Equations Using the Laplace Transform . . . 237
6.5 Response of First-Order Systems . . . 239
6.6 Review of Complex Variables . . . 243
6.7 Second-Order Systems . . . 244
6.8 Free Response of Undamped Second-Order Systems . . . 246
6.9 Free Response of Damped Second-Order Systems . . . 250
6.10 Underdamped Systems . . . 251
6.11 Damping Estimation by Logarithmic Decrement . . . 255
6.12 Response to an Impulsive Force . . . 258
6.12.1 Response to Multiple Impulses . . . 261
6.13 Step Response . . . 263
6.14 Response to General Excitations—Convolution Integral . . . 271
6.15 Time-Domain vs. Frequency-Domain Analysis . . . 274
6.16 Response to Harmonic Excitation . . . 278
6.16.1 General Formulation for Second-Order Systems . . . 278
6.16.2 Quality Factor . . . 280
6.16.3 Phase Angle . . . 281
6.17 Resonance . . . 282
6.18 Transmitted Force . . . 285
6.19 Base Excitation . . . 287
6.20 Harmonic Excitation Due to Imbalances and Eccentricity . . . 290
6.20.1 Rotating Imbalances . . . 290
6.20.2 Whirling of Rotating Shafts . . . 291
6.21 Bibliography . . . 294
6.22 Problems . . . 294

7 Response of Multi-Degrees-of-Freedom Systems 303
7.1 Introduction . . . 303
7.2 Modeling of Multi-Degrees-of-Freedom Systems . . . 303
7.2.1 Definition of Sign Definiteness . . . 304
7.3 Coupling . . . 312
7.4 Free Motion of Undamped Multi-Degrees-of-Freedom Systems . . . 312
7.4.1 Natural Frequencies . . . 314
7.4.2 Modal Vectors . . . 315
7.4.3 Normalization of Modal Vectors . . . 318
7.4.4 General Form of the Free Response . . . 320
7.5 Solving for the Natural Frequencies and Modal Vectors . . . 321
7.6 Beat Phenomenon . . . 322
7.7 Unrestrained Motion and Rigid Body Modes . . . 327
7.8 Orthogonality of the Modal Vectors . . . 331
7.9 Expansion Theorem . . . 332
7.9.1 Expansion by Sets of Functions . . . 333
7.9.2 Expansion by Geometric Vectors . . . 334
7.9.3 Expansion by Modal Vectors . . . 335
7.10 Modal Equations of Motion and Response . . . 336
7.10.1 Modal Response . . . 337
7.10.2 Modal Response Using Homogeneous and Particular Solution Approach . . . 339
7.10.3 Response of Modes with Zero Eigenvalues . . . 340
7.10.4 Response to Impulsive Loading . . . 341
7.11 Mode Participation and Isolation . . . 345

7.11.1 Mode Participation . . . 345
7.11.2 Mode Isolation—Mode Control . . . 347
7.12 Approximate Approach for Damped Systems . . . 348
7.13 Response to Harmonic Excitation . . . 352
7.13.1 General Formulation . . . 352
7.13.2 Special Case: Two Degrees of Freedom System . . . 353
7.13.3 Modal Analysis . . . 355
7.14 Vibration Reducing Devices . . . 356
7.14.1 Motion Amplitude before Vibration Reducer Is Added . . . 357
7.14.2 Undamped Vibration Absorbers . . . 358
7.14.3 Tuned Mass Dampers . . . 362
7.15 First-Order Systems . . . 364
7.16 Numerical Integration . . . 365
7.17 Bibliography . . . 367
7.18 Problems . . . 368

8 Analytical Mechanics 375
8.1 Introduction . . . 375
8.2 Generalized Coordinates and Constraints . . . 375
8.2.1 Generalized Coordinates . . . 376
8.2.2 Constraints and Constraint Forces . . . 377
8.3 Velocity Representation . . . 381
8.3.1 Representation by Generalized Velocities . . . 381
8.3.2 Representation by Generalized Speeds (Quasi-Velocities) . . . 382
8.3.3 Relationship between Generalized Velocities and Generalized Speeds . . . 382
8.4 Virtual Displacements and Virtual Work . . . 388
8.5 Virtual Displacements and Virtual Work for Rigid Bodies . . . 390
8.6 Generalized Forces . . . 394
8.7 Principle of Virtual Work for Static Equilibrium . . . 399
8.8 D'Alembert's Principle . . . 401
8.8.1 General Formulation . . . 401
8.8.2 Extension to Rigid Bodies . . . 402
8.8.3 Using D'Alembert's Principle to Obtain Equations of Motion . . . 404
8.9 Hamilton's Principle . . . 408
8.10 Lagrange's Equations . . . 411
8.11 Constrained Systems . . . 418
8.11.1 Lagrange Multiplier Method . . . 419
8.11.2 Constraint Relaxation Method . . . 421
8.12 Kane's Equations . . . 422
8.13 Natural and Nonnatural Systems, Equilibrium . . . 426
8.14 Small Motions about Equilibrium . . . 431
8.15 Rayleigh's Dissipation Function . . . 437
8.16 Generalized Momentum, First Integrals . . . 439
8.17 Impulsive Motion . . . 440
8.17.1 Impulsive Excitation in Lagrangian Mechanics . . . 440
8.17.2 Impulse-Momentum Relationships for Kane's Equations . . . 441
8.18 Bibliography . . . 444
8.19 Problems . . . 444

9 Three-Dimensional Kinematics of Rigid Bodies 459
9.1 Introduction . . . 459
9.2 Basic Kinematics of Rigid Bodies . . . 459
9.2.1 Pure Translation . . . 460
9.2.2 Pure Rotation . . . 460
9.2.3 Combined Translation and Rotation . . . 465
9.3 Euler Angles . . . 467
9.3.1 Euler Angle Sequences . . . 467
9.3.2 Angular Velocity . . . 470
9.3.3 Angular Acceleration . . . 472
9.4 Axisymmetric Bodies . . . 475
9.5 Rolling . . . 478
9.5.1 Disk Originally Lying Flat . . . 480
9.5.2 Beginning with the Disk as Vertical . . . 483
9.5.3 Steady Precession . . . 485
9.6 Orientation Change by Successive Rotations . . . 488
9.7 Interconnections . . . 490
9.7.1 Basic Types of Joints . . . 490
9.7.2 Combined Sliding and Rotation . . . 492
9.7.3 Universal Joints . . . 493
9.8 4×4 Matrix Description of a General Transformation . . . 496
9.9 Euler Parameters . . . 499
9.9.1 Transformation Matrix in Terms of Euler Parameters . . . 501
9.9.2 Relating the Euler Parameters to Angular Velocities . . . 502
9.9.3 Relating Euler Parameters to the Transformation Matrix . . . 503
9.9.4 Relating Euler Parameters to the Euler Angles . . . 504
9.10 Rodrigues Parameters . . . 506
9.10.1 Screw Algebra and Rodrigues Parameters . . . 508
9.10.2 Rotation of a Body . . . 512
9.10.3 Finding the Screw Axis and Rotation Angle . . . 513
9.11 Bibliography . . . 517
9.12 Problems . . . 517

10 Mass Moments of Inertia 527
10.1 Introduction . . . 527
10.2 Center of Mass . . . 527
10.3 Mass Moment of Inertia . . . 528
10.3.1 Calculation of the Mass Moments and Products of Inertia . . . 530
10.4 Transformation Properties of the Inertia Matrix . . . 536
10.4.1 Translation of Coordinates . . . 536
10.4.2 Rotation of Coordinate Axes . . . 538
10.5 Principal Moments of Inertia . . . 542
10.5.1 Special Case: Repeated Principal Moments of Inertia . . . 543
10.6 Bibliography . . . 546
10.7 Problems . . . 546

11 Dynamics of Three-Dimensional Rigid Body Motion 553
11.1 Introduction . . . 553
11.2 Linear and Angular Momentum . . . 553
11.3 Transformation Properties of Angular Momentum . . . 557
11.3.1 Translation of Coordinates . . . 557

11.3.2 Rotation of Coordinates . . . 558
11.4 General Describing Equations . . . 562
11.4.1 Resultant Force and Moment . . . 563
11.4.2 Force and Moment Balances . . . 564
11.4.3 Moment Balance about an Arbitrary Different Point . . . 564
11.5 Description in Terms of Body Fixed Coordinates . . . 565
11.6 Angular Momentum Balance for Axisymmetric Bodies . . . 569
11.6.1 Modified Euler's Equations . . . 569
11.6.2 Expressing the Rotational Equations Using Components of Shape Frame . . . 571
11.7 Stability Analysis of Rotational Motion . . . 577
11.8 Steady Precession of a Rolling Disk . . . 579
11.9 Rotation about a Fixed Axis . . . 584
11.10 Impulse and Momentum . . . 588
11.11 Energy and Work . . . 590
11.11.1 Kinetic Energy . . . 590
11.11.2 Work and Conservation of Energy . . . 592
11.12 Analytical Equations for Rigid Bodies . . . 594
11.12.1 Euler Angles as Generalized Coordinates . . . 594
11.12.2 Lagrange's and Kane's Equations . . . 595
11.13 Torque-Free Motion of Axisymmetric Bodies . . . 602
11.13.1 Integrals of the Motion . . . 602
11.13.2 Body and Space Cones . . . 604
11.13.3 Direct and Retrograde Precession . . . 606
11.13.4 Energy Dissipation and Nutational Instability . . . 607
11.14 Bibliography . . . 611
11.15 Problems . . . 611

12 Vehicle Dynamics—Basic Loads and Longitudinal Motions 623
12.1 Introduction . . . 623
12.2 Vehicle Coordinate Systems and Nomenclature . . . 623
12.3 Loads on Vehicles . . . 625
12.4 Acceleration . . . 626
12.4.1 Acceleration Analysis . . . 627
12.4.2 Maximum Acceleration . . . 628
12.5 Power . . . 630
12.5.1 Constant Power Approximation . . . 632
12.6 More Advanced Model Including Wheel Inertia . . . 634
12.7 Braking . . . 637
12.7.1 Brake Effects . . . 637
12.7.2 Force Analysis . . . 638
12.7.3 Brake Proportioning . . . 641
12.7.4 Anti-Lock Brake Systems . . . 644
12.8 Rollover and Lateral Instability . . . 647
12.8.1 Simple Combined Lateral and Roll Analysis . . . 650
12.8.2 Critical Sliding Velocity . . . 650
12.9 Weight Shift and Statical Indeterminacy . . . 656
12.9.1 Statically Indeterminate Systems . . . 656
12.9.2 An Approximate Method for Calculating Wheel Loads . . . 657
12.9.3 Calculation of Center of Mass Location . . . 662
12.10 Bibliography . . . 665

12.11 Problems . . . 665

13 Vehicle Dynamics—Tire and Aerodynamic Forces **671**
13.1 Introduction . . . 671
13.2 Tires . . . 671
13.2.1 Tire Terminology . . . 672
13.2.2 Tire Components . . . 673
13.3 Tire Forces . . . 674
13.3.1 Resultant Tire Forces and Moments . . . 677
13.4 Lateral Forces and Tire Slip . . . 678
13.5 Tire Torques . . . 682
13.5.1 Aligning Torque . . . 683
13.5.2 Overturning Moment . . . 685
13.6 Slip Ratio and Longitudinal Tire Forces . . . 685
13.6.1 Wheel Slip . . . 685
13.6.2 Tractive and Lateral Tire Forces . . . 686
13.6.3 More Comprehensive Tire Models . . . 687
13.7 Rolling Resistance . . . 687
13.7.1 Factors Affecting Rolling Resistance . . . 691
13.7.2 Induced Drag . . . 692
13.7.3 Rolling Resistance Models . . . 693
13.8 Camber . . . 694
13.9 Other Tire Effects . . . 695
13.9.1 Pressure . . . 696
13.9.2 Temperature . . . 697
13.9.3 Speed . . . 697
13.9.4 Conicity and Ply Steer . . . 697
13.10 Summary of Tire Force Effects . . . 698
13.11 Nondimensional Analysis of Tire Behavior . . . 699
13.12 Aerodynamic Forces . . . 700
13.12.1 Calculation of Aerodynamic Coefficients from Test Data . . . 704
13.13 Bibliography . . . 706
13.14 Problems . . . 707

14 Vehicle Dynamics—Lateral Stability **711**
14.1 Introduction . . . 711
14.2 Kinematics—Steer Angle Definitions . . . 711
14.3 Wheel Loads and Slip Angles . . . 717
14.3.1 Free-Body Diagram and Slip Angles . . . 717
14.3.2 Understeer Gradient and Critical Speed . . . 720
14.3.3 Bundorf Compliances . . . 723
14.3.4 Lateral Acceleration Gain . . . 723
14.4 Slip Angle Kinematics . . . 725
14.5 Transient Motion Equations . . . 728
14.6 Response . . . 731
14.6.1 Numerical Integration of the Stability Equations . . . 732
14.7 Eigenvalue Analysis . . . 735
14.8 Mass-Spring-Damper Analogy . . . 739
14.9 Steady-State Response . . . 742
14.10 Yaw Velocity Gain and Curvature Response . . . 745
14.11 Tangent Speed and Hydroplaning . . . 746

14.12 Neutral Steer Point . . . 748
14.13 Modeling the Driver . . . 749
14.14 Electronic Stability Control . . . 751
14.15 Which Wheels Will Slide First? . . . 756
14.16 Bibliography . . . 761
14.17 Problems . . . 762

15 Vehicle Dynamics—Bounce, Pitch, and Roll **763**
15.1 Introduction . . . 763
15.2 Sources of Excitation . . . 764
15.2.1 Ride Quality and Human Response to Vibration . . . 765
15.3 Unsprung vs. Sprung Mass . . . 766
15.4 Simple Suspension Models . . . 766
15.5 Quarter-Car Model . . . 770
15.5.1 Single-Degree-of-Freedom Model . . . 770
15.5.2 Change in Natural Frequency and Damping Factor Due to Payload Weight . . . 774
15.5.3 Wheel Hop . . . 774
15.5.4 Two-Degrees-of-Freedom Model . . . 775
15.6 Pitch and Bounce Motions . . . 778
15.6.1 Equations of Motion . . . 778
15.6.2 Analysis of the Equations of Motion and Response . . . 780
15.7 Olley Criteria . . . 785
15.8 Response to Harmonic Excitation . . . 786
15.8.1 Modeling an Uneven Road . . . 786
15.8.2 Frequency Response for a Single-Degree-of-Freedom Model . . . 788
15.8.3 Frequency Response for Two-Degrees-of-Freedom Models . . . 791
15.9 Roll Dynamics . . . 794
15.10 Roll Center Analysis . . . 798
15.11 Lateral Force Reduction Due to Weight Shift . . . 801
15.12 Roll Axis . . . 803
15.12.1 Design for Cornering . . . 806
15.13 Introduction to Suspension Systems . . . 807
15.14 Suspension System Terminology and Geometry . . . 807
15.15 Axle Suspensions . . . 811
15.16 Independent Suspensions . . . 814
15.17 Roll Center Construction . . . 817
15.18 Jacking . . . 820
15.19 Scrub . . . 821
15.20 Anti-Roll Bar . . . 822
15.21 Force Analysis for Anti-Squat and Anti-Dive . . . 824
15.21.1 Independent Suspensions and Rear Wheel Drive . . . 828
15.21.2 Front Wheel Drive . . . 828
15.21.3 Four Wheel Drive . . . 829
15.21.4 Anti-Dive for Braking . . . 829
15.22 Bibliography . . . 830
15.23 Problems . . . 831

16 Appendix—Common Inertia Properties **835**

Index **839**