

CONTENTS

Preface	vii
1 Geometric Setting	1
1.1 VECTORS AND EUCLIDEAN POINT SPACE	2
1.1.1 Vectors	2
1.1.2 Euclidean Point Space	6
1.1.3 Summary	8
1.2 TENSORS	8
1.2.1 First-Order Tensors and Vectors	8
1.2.2 Second-Order Tensors	11
1.2.3 Cross Products, Triple Products, and Determinants	15
1.2.4 Orthogonal Tensors	20
1.2.5 Invariants of a Tensor	21
1.2.6 Derivatives of Tensor-Valued Functions	24
1.2.7 Summary	27
	xi

2 Kinematics I: The Calculus of Motion	29
2.1 BODIES, MOTIONS, AND DEFORMATIONS	29
2.1.1 Deformation	32
2.1.2 Examples of Motions	33
2.1.3 Summary	36
2.2 DERIVATIVES OF MOTION	36
2.2.1 Time Derivatives	37
2.2.2 Derivatives With Respect to Position	38
2.2.3 The Deformation Gradient	40
2.2.4 Summary	42
2.3 PATHLINES, STREAMLINES, AND STREAKLINES	43
2.3.1 Three Types of Arc	43
2.3.2 An Example	45
2.3.3 Summary	49
2.4 INTEGRALS UNDER MOTION	49
2.4.1 Arc, Surface, and Volume Integrals	49
2.4.2 Reynolds Transport Theorem	55
2.4.3 Summary	57
3 Kinematics II: Strain and its Rates	59
3.1 STRAIN	59
3.1.1 Symmetric Tensors	60
3.1.2 Polar Decomposition and the Deformation Gradient	64
3.1.3 Examples	66
3.1.4 Cauchy–Green and Strain Tensors	68
3.1.5 Strain Invariants	70
3.1.6 Summary	71
3.2 INFINITESIMAL STRAIN	72
3.2.1 The Infinitesimal Strain Tensor	72
3.2.2 Summary	75
3.3 STRAIN RATES	75
3.3.1 Stretching and Spin Tensors	76
3.3.2 Skew Tensors, Spin, and Vorticity	79
3.3.3 Summary	84
3.4 VORTICITY AND CIRCULATION	84
3.4.1 Circulation	84
3.4.2 Summary	88

3.5	OBSERVER TRANSFORMATIONS	89
3.5.1	Changes in Frame of Reference	89
3.5.2	Summary	95
4	Balance Laws	97
4.1	MASS BALANCE	98
4.1.1	Local Forms of Mass Balance	99
4.1.2	Summary	102
4.2	MOMENTUM BALANCE	102
4.2.1	Analysis of Stress	104
4.2.2	Inertial Frames of Reference	110
4.2.3	Momentum Balance in Referential Coordinates	113
4.2.4	Summary	114
4.3	ANGULAR MOMENTUM BALANCE	115
4.3.1	Symmetry of the Stress Tensor	117
4.3.2	Summary	118
4.4	ENERGY BALANCE	119
4.4.1	Thermal Energy Balance	122
4.4.2	Summary	124
4.5	ENTROPY INEQUALITY	124
4.5.1	Motivation	125
4.5.2	Clausius–Duhem Inequality	126
4.5.3	Summary	127
4.6	JUMP CONDITIONS	127
4.6.1	Singular Surfaces	129
4.6.2	Localization	132
4.6.3	Summary	135
5	Constitutive Relations: Examples of Mathematical Models	137
5.1	HEAT TRANSFER	138
5.1.1	Properties of the Heat Equation	140
5.1.2	Summary	142
5.2	POTENTIAL THEORY	143
5.2.1	Motivation	143
5.2.2	Boundary Conditions	144
5.2.3	Uniqueness of Solutions to the Poisson Equation	146
5.2.4	Maximum Principle	147
5.2.5	Mean Value Property	150

5.2.6	Summary	151
5.3	FLUID MECHANICS	152
5.3.1	Ideal Fluids	152
5.3.2	An Ideal Fluid in a Rotating Frame of Reference	154
5.3.3	Acoustics	155
5.3.4	Incompressible Newtonian Fluids	158
5.3.5	Stokes Flow	159
5.3.6	Summary	163
5.4	SOLID MECHANICS	164
5.4.1	Static Displacements	164
5.4.2	Elastic Waves	167
5.4.3	Summary	170
6	Constitutive Theory	173
6.1	CONCEPTUAL SETTING	174
6.1.1	The Need to Close the System	174
6.1.2	Summary	176
6.2	DETERMINISM AND EQUIPRESENCE	177
6.2.1	Determinism	177
6.2.2	Equipresence	177
6.2.3	Summary	178
6.3	OBJECTIVITY	179
6.3.1	Reducing Functional Dependencies	180
6.3.2	Summary	182
6.4	SYMMETRY	183
6.4.1	Changes in Reference Configuration	183
6.4.2	Symmetry Groups	186
6.4.3	Classification of Materials	189
6.4.4	Implications for Thermoviscous Fluids	193
6.4.5	Summary	193
6.5	ADMISSIBILITY	194
6.5.1	Implications of the Entropy Inequality	195
6.5.2	Analysis of Equilibrium	197
6.5.3	Linear, Isotropic, Thermoelastic Solids	199
6.5.4	Summary	202
7	Multiconstituent Continua	203
7.1	CONSTITUENTS	204

7.1.1	Configurations and Motions	204
7.1.2	Volume Fractions and Densities	206
7.1.3	Summary	208
7.2	MULTICONSTITUENT BALANCE LAWS	209
7.2.1	Multiconstituent Mass Balance	210
7.2.2	Multiconstituent Momentum Balance	212
7.2.3	Multiconstituent Angular Momentum Balance	214
7.2.4	Multiconstituent Energy Balance	215
7.2.5	Multiconstituent Entropy Inequality	216
7.2.6	Isothermal, Nonreacting Multiphase Mixtures	217
7.2.7	Summary	219
7.3	FLUID FLOW IN A POROUS SOLID	220
7.3.1	Modeling Assumptions for Porous Media	221
7.3.2	Balance Laws for the Fluid and Solid Phases	223
7.3.3	Equilibrium Constraints	225
7.3.4	Linear Extensions From Equilibrium	226
7.3.5	Commentary	228
7.3.6	Potential Formulation of Darcy's Law	229
7.3.7	Summary	233
7.4	DIFFUSION IN A BINARY FLUID MIXTURE	234
7.4.1	Modeling Assumptions for Binary Diffusion	235
7.4.2	Balance Laws for the Two Species	235
7.4.3	Constitutive Relationships for Diffusion	236
7.4.4	Modeling Solute Transport	239
7.4.5	Summary	242
A	Guide to Notation	243
A.1	GENERAL CONVENTIONS	243
A.2	LETTERS RESERVED FOR DEDICATED USES	244
A.3	SPECIAL SYMBOLS	245
B	Vector Integral Theorems	247
B.1	STOKES'S THEOREM	248
B.2	THE DIVERGENCE THEOREM	249
B.3	THE CHANGE-OF-VARIABLES THEOREM	252
C	Hints and Solutions to Exercises	253
References	265	
Index	269	