

# Contents

<i>Preface</i>	<i>page xv</i>
<b>1 Introduction</b> . . . . .	1
1.1 Purpose and Motivation	1
1.2 Problem Statement	2
1.3 Scope of the Book	3
1.3.1 Systems Theory	3
1.3.2 Control Theory	5
1.3.3 Aerospace Applications	6
1.4 Examples	6
1.4.1 Transoceanic Jetliner Flight	6
1.4.2 Intelligence, Surveillance, and Reconnaissance with Unmanned Aerial Vehicle	6
1.4.3 Homing Guidance of Heat-Seeking Missile	7
1.4.4 Spacecraft Orbital Maneuvers	9
1.4.5 Interplanetary Travel	9
1.5 Content of the Book	12
1.6 Summary of Key Results	12
1.7 Bibliographic Notes for Further Reading	12
1.8 Homework Problems	12
<b>2 Deterministic Systems Theory</b> . . . . .	14
2.1 Linear Dynamic Systems	14
2.1.1 System Linearization	14
2.1.2 Properties of Linear Dynamic Systems	21
2.2 Observability	26
2.3 Time Invariant Systems	30
2.3.1 Stability of Linear Time Invariant Systems	33
2.3.2 BIBO Stability of Linear Time Invariant Systems	34
2.3.3 Observability of Linear Time Invariant Systems	35
2.4 The Method of Adjoint	37
2.5 Controllability and Duality	39
2.6 Summary of Key Results	42

2.7	Bibliographic Notes for Further Reading	42
2.8	Homework Problems	42
<b>3</b>	<b>Stochastic Systems Theory</b>	<b>48</b>
3.1	Probability Spaces	49
3.2	Random Variables and Distributions	51
3.3	Expected Value and Characteristic Function	57
3.4	Independence and Correlation	59
3.5	The Gaussian Distribution	59
3.6	Random Processes	62
3.7	Gauss–Markov Processes	67
3.8	Linear Gauss–Markov Models	69
3.9	Summary of Key Results	72
3.10	Bibliographic Notes for Further Reading	72
3.11	Homework Problems	72
<b>4</b>	<b>Navigation</b>	<b>78</b>
4.1	Position Fixing: The Ideal Case	78
4.2	Position Fixing: Error Analysis	80
4.3	Position Fixing: Redundant Measurements	84
4.4	Examples of Fixes	87
4.5	Inertial Navigation	93
4.5.1	Inertially Stabilized Inertial Navigation Systems	94
4.5.2	Strapped Down Inertial Navigation Systems	95
4.6	Asymptotic Observers	98
4.7	The Kalman Filter	100
4.8	The Extended Kalman Filter	105
4.9	Clock Corrections	107
4.10	Navigation Hardware	108
4.11	Summary of Key Results	111
4.12	Bibliographic Notes for Further Reading	111
4.13	Homework Problems	112
<b>5</b>	<b>Homing Guidance</b>	<b>118</b>
5.1	Fundamentals of Homing	118
5.2	Pursuit Guidance	121
5.2.1	Terminal Phase Analysis	123
5.2.2	Approximate Miss Distance Analysis	124
5.2.3	Exact Miss Distance Analysis	126
5.3	Fixed Lead Guidance	129
5.4	Constant Bearing Guidance	129
5.5	Proportional Navigation	129
5.6	Linearized Proportional Navigation	130
5.6.1	Miss due to Launch Error	135
5.6.2	Miss due to Step Target Acceleration	138
5.6.3	Miss due to Target Sinusoidal Motion	139

5.6.4	Miss due to Noise	140
5.6.5	Use of Power Series Solution	141
5.7	Beam Rider Guidance	144
5.8	Summary of Key Results	146
5.9	Bibliographic Notes for Further Reading	146
5.10	Homework Problems	147
<b>6</b>	<b>Ballistic Guidance</b>	<b>150</b>
6.1	The Restricted Two-Body Problem	150
6.2	The Two-Dimensional Hit Equation	154
6.3	In-Plane Error Analysis	158
6.4	Three-Dimensional Error Analysis	163
6.4.1	Actual Flight Time Approximation	166
6.4.2	Down-Range Miss Distance, $M_{DR}$	167
6.4.3	Cross-Range Miss Distance, $M_{CR}$	169
6.5	Effects of the Earth's Rotation	170
6.6	Effects of Earth's Oblateness and Geophysical Uncertainties	174
6.6.1	Effects of Other Perturbations	175
6.7	General Solution of Ballistic Guidance Problems	175
6.7.1	General Framework	175
6.7.2	Problem Formulation	176
6.7.3	Examples	176
6.7.4	Targeting	179
6.7.5	Miss Analysis	181
6.8	Summary of Key Results	184
6.9	Bibliographic Notes for Further Reading	184
6.10	Homework Problems	185
<b>7</b>	<b>Midcourse Guidance</b>	<b>187</b>
7.1	Velocity-to-Be-Gained Guidance	188
7.1.1	Velocity-to-Be-Gained Guidance with Unlimited Thrust	189
7.1.2	Velocity-to-Be-Gained Guidance with Limited Thrust	189
7.2	Guidance by State Feedback	191
7.3	Combined Navigation and Guidance	193
7.4	Summary of Key Results	196
7.5	Bibliographic Notes for Further Reading	197
7.6	Homework Problems	197
<b>8</b>	<b>Optimization</b>	<b>199</b>
8.1	Unconstrained Optimization on $\mathbb{R}^n$	200
8.2	Constrained Optimization on $\mathbb{R}^n$	205
8.2.1	Lagrange Multipliers	206
8.2.2	Second-Order Conditions	208
8.3	Inequality Constraints on $\mathbb{R}^n$	214
8.4	Optimal Control of Discrete-Time Systems	215
8.5	Summary of Key Results	217

8.6	Bibliographic Notes for Further Reading	218
8.7	Homework Problems	218
<b>9</b>	<b>Optimal Guidance</b> . . . . .	<b>222</b>
9.1	Problem Formulation	222
9.2	Examples	225
9.3	Optimal Control without Control Constraints	228
9.4	The Maximum Principle	233
9.4.1	Greed	235
9.4.2	The Transversality Conditions	236
9.4.3	Target Sets	237
9.4.4	Time-Optimal Control of Double Integrator	238
9.4.5	Optimal Evasion through Jinking	241
9.5	Dynamic Programming	246
9.5.1	Motivational Example: Dynamic Programming	246
9.5.2	The Principle of Optimality	248
9.5.3	Backward Dynamic Programming	249
9.5.4	Continuous-Time Dynamic Programming	250
9.5.5	The Linear Quadratic Regulator	252
9.5.6	The Linear Quadratic Gaussian Regulator	255
9.5.7	Relationship between the Maximum Principle and Dynamic Programming	256
9.5.8	The Hamilton–Jacobi–Bellman Equation	258
9.5.9	Dynamic Programming Summary	258
9.6	The Maximum Principle and Dynamic Programming	259
9.7	Summary of Key Results	261
9.8	Bibliographic Notes for Further Reading	262
9.9	Homework Problems	262
<b>10</b>	<b>Introduction to Differential Games</b> . . . . .	<b>269</b>
10.1	Taxonomy of Two-Player Games	269
10.2	Example of a Simple Pursuit Game: Two-Player Football Scrimmage	272
10.2.1	Modeling	272
10.2.2	Analysis	272
10.2.3	The Apollonius Circle Theorem	273
10.2.4	Solution to the Football Two-Player Scrimmage Problem	275
10.3	The Bellman–Isaacs Equation	275
10.4	The Homicidal Chauffeur: Modeling	276
10.5	The Homicidal Chauffeur: Features of the Solution	280
10.6	A Game-Theoretic View of Proportional Navigation	281
10.7	Summary of Key Results	285
10.8	Bibliographic Notes for Further Reading	285
10.9	Homework Problems	285
	<b>Epilogue</b> . . . . .	<b>288</b>

APPENDIX A: Useful Definitions and Mathematical Results . . . . .	295
A.1 Results from Topology	295
A.2 Results from Linear Algebra	297
A.3 Taylor's Theorem	300
A.4 Newton's Method	301
A.5 The Implicit Function Theorem	301
<i>Bibliography</i>	305
<i>Index</i>	309