

# Contents

Preface

page xi

|                                                                  |           |
|------------------------------------------------------------------|-----------|
| <b>1 Aircraft Structural Components and Loads</b>                | <b>1</b>  |
| 1.0.1 Fuselage                                                   | 1         |
| 1.0.2 Wing                                                       | 2         |
| 1.1 Elements of Aerodynamic Forces                               | 3         |
| 1.2 Level Flight                                                 | 4         |
| 1.3 Load Factor                                                  | 5         |
| 1.4 Maneuvers                                                    | 5         |
| 1.5 Gust Load                                                    | 6         |
| 1.6 $V$ - $n$ Diagram                                            | 7         |
| 1.7 Proof Load and Ultimate Load                                 | 8         |
| 1.8 Optimization                                                 | 8         |
| Further Reading                                                  | 9         |
| Exercises                                                        | 9         |
| <b>2 Elements of Elasticity</b>                                  | <b>11</b> |
| 2.1 Traction Vector                                              | 11        |
| 2.1.1 External and Internal Forces                               | 11        |
| 2.2 Stress Components                                            | 13        |
| 2.2.1 Example: Traction Vector on a Plane                        | 14        |
| 2.3 Equilibrium Equations                                        | 15        |
| 2.3.1 Force Equilibrium                                          | 15        |
| 2.3.2 Moment Equilibrium                                         | 16        |
| 2.3.3 Traction Boundary Conditions                               | 17        |
| 2.4 Plane Stress                                                 | 17        |
| 2.4.1 Curved Panels                                              | 18        |
| 2.5 Transformation of Vectors                                    | 18        |
| 2.6 Transformation of Tensors                                    | 19        |
| 2.6.1 Principal Stresses                                         | 21        |
| 2.6.2 Maximum Shear Stress                                       | 23        |
| 2.6.3 Mohr's Circle                                              | 23        |
| 2.6.4 Example: Principal Stresses and the Maximum Shear Stresses | 24        |

|          |                                                      |           |
|----------|------------------------------------------------------|-----------|
| 2.7      | Principal Stresses in Three Dimensions               | 25        |
| 2.7.1    | Example: Eigenvalues and Eigenvectors                | 27        |
| 2.8      | Displacement and Strain                              | 28        |
| 2.8.1    | Plane Strain                                         | 32        |
| 2.8.2    | Strain Transformation                                | 32        |
| 2.8.3    | Displacement Boundary Conditions                     | 33        |
| 2.8.4    | Compatibility Equations                              | 33        |
| 2.9      | Generalized Hooke's Law                              | 33        |
| 2.9.1    | Hooke's Law for Plane Stress                         | 34        |
| 2.9.2    | Hooke's Law for Plane Strain                         | 34        |
| 2.9.3    | Example: Cantilever Beam under Plane Stress          | 35        |
| 2.10     | Thermal Expansion                                    | 38        |
| 2.10.1   | Example: Heated Bar                                  | 39        |
| 2.11     | Saint Venant's Principle                             | 39        |
| 2.12     | Anisotropic Materials                                | 40        |
| 2.13     | Composite Materials                                  | 40        |
| 2.13.1   | Lamina Properties                                    | 41        |
| 2.13.2   | Laminate Properties                                  | 43        |
| 2.14     | Plasticity in Polycrystalline Metals                 | 44        |
| 2.15     | Commonly Used Aircraft Materials                     | 45        |
|          | Further Reading                                      | 46        |
|          | Exercises                                            | 46        |
| <b>3</b> | <b>Energy Methods</b>                                | <b>50</b> |
| 3.1      | Strain Energy and Complementary Strain Energy        | 52        |
| 3.1.1    | Flexibility and Stiffness Coefficient                | 53        |
| 3.2      | Strain Energy Density                                | 55        |
| 3.2.1    | Voigt Notation                                       | 56        |
| 3.2.2    | Uniform Bar Subjected to an Axial Tension            | 56        |
| 3.2.3    | Circular Shaft Subjected to a Torque                 | 57        |
| 3.2.4    | Beam Subjected to a Moment                           | 58        |
| 3.3      | Two-bar Truss                                        | 58        |
| 3.3.1    | Solution Without Using Energy                        | 59        |
| 3.3.2    | Virtual Force Method                                 | 60        |
| 3.3.3    | Castigliano's First Theorem                          | 60        |
| 3.3.4    | Unit Load Method                                     | 61        |
| 3.3.5    | Virtual Displacement Method                          | 61        |
| 3.3.6    | Castigliano's Second Theorem                         | 62        |
| 3.3.7    | Unit Displacement Method                             | 62        |
| 3.3.8    | Minimum Total Complementary Potential Energy Theorem | 63        |
| 3.3.9    | Minimum Total Potential Energy Theorem               | 63        |
| 3.3.10   | Dummy Load and Dummy Displacements                   | 64        |
| 3.4      | Statically Indeterminate Problems                    | 64        |
| 3.4.1    | Example: Statically Indeterminate Torsion            | 65        |
| 3.4.2    | Example: Beam under a Concentrated Load              | 66        |

|          |                                                                                                   |            |
|----------|---------------------------------------------------------------------------------------------------|------------|
| 3.4.3    | Example: Axial Extension Calculation of a Variable Cross-section Bar by the Finite Element Method | 67         |
| 3.4.4    | Example: Frame Deflection                                                                         | 70         |
| 3.5      | Example: Bulkhead Subjected to Moments                                                            | 70         |
| 3.6      | Example: Shear Correction Factor                                                                  | 72         |
| 3.7      | Reciprocity Theorem                                                                               | 73         |
|          | Further Reading                                                                                   | 74         |
|          | Exercises                                                                                         | 74         |
| <b>4</b> | <b>Torsion</b>                                                                                    | <b>79</b>  |
| 4.1      | Solid Circular Shaft                                                                              | 79         |
| 4.2      | Thin-walled Circular Tube                                                                         | 80         |
| 4.3      | Single-Cell Tube                                                                                  | 81         |
| 4.3.1    | Example: A Rectangular Tube                                                                       | 83         |
| 4.4      | Multi-cell Tubes                                                                                  | 83         |
| 4.4.1    | Example: Three-cell Tube                                                                          | 84         |
| 4.5      | Open Thin-walled Tubes                                                                            | 85         |
| 4.5.1    | Example: Circular Tube                                                                            | 87         |
| 4.6      | Warping of the Cross Section                                                                      | 87         |
| 4.6.1    | Example: Warping in a Single-cell Tube                                                            | 89         |
| 4.6.2    | Warping in Open Tubes                                                                             | 91         |
| 4.6.3    | Example: Split Circular Tube                                                                      | 91         |
| 4.7      | Induced Torque due to Varying Warp                                                                | 92         |
| 4.7.1    | Example: Channel Section                                                                          | 94         |
| 4.7.2    | Example: Split Circular Tube                                                                      | 96         |
|          | Further Reading                                                                                   | 96         |
|          | Exercises                                                                                         | 97         |
| <b>5</b> | <b>Bending</b>                                                                                    | <b>100</b> |
| 5.0.1    | Principal Axes of a Section                                                                       | 103        |
| 5.0.2    | Example: Angle Section                                                                            | 103        |
| 5.1      | Equilibrium Equations                                                                             | 106        |
| 5.1.1    | Example: Elliptic Lift Distribution                                                               | 107        |
| 5.1.2    | Concentrated Forces and Moments                                                                   | 108        |
| 5.1.3    | Example: Concentrated Force and Moment                                                            | 109        |
| 5.2      | Shear Stresses in Beams                                                                           | 111        |
| 5.2.1    | Open-cell Tubes                                                                                   | 111        |
| 5.2.2    | Example: Channel Section                                                                          | 112        |
| 5.2.3    | Center of Shear                                                                                   | 113        |
| 5.2.4    | Example: Split Circular Beam                                                                      | 114        |
| 5.2.5    | Center of Shear and the Center of Twist                                                           | 115        |
| 5.3      | Multi-cell Tubes                                                                                  | 116        |
| 5.3.1    | Shear Flow Transfer at Wall Intersections                                                         | 117        |
| 5.3.2    | Example: Two-cell Tube                                                                            | 117        |
| 5.4      | Sheet-stringer Construction                                                                       | 121        |
| 5.4.1    | Example: Open Cell                                                                                | 122        |
| 5.4.2    | Example: A Two-cell Beam under Boom Approximation                                                 | 123        |

|          |                                                                   |            |
|----------|-------------------------------------------------------------------|------------|
| 5.5      | Shear Lag                                                         | 124        |
| 5.6      | Combined Bending and Torsion                                      | 126        |
| 5.6.1    | Example: Single-cell Beam                                         | 126        |
| 5.7      | Analysis of Tapered Beams                                         | 128        |
| 5.7.1    | Quadrilateral Shear Panels                                        | 128        |
| 5.7.2    | Shear Flow due to Bending in Tapered Beams                        | 129        |
|          | Further Reading                                                   | 131        |
|          | Exercises                                                         | 131        |
| <b>6</b> | <b>Stability of Structures</b>                                    | <b>135</b> |
| 6.0.3    | Spring-supported Vertical Bar                                     | 135        |
| 6.1      | Buckling of a Beam                                                | 136        |
| 6.1.1    | Critical Stresses                                                 | 138        |
| 6.2      | Bending under an Eccentric Load                                   | 138        |
| 6.3      | Buckling of Imperfect Beams                                       | 140        |
| 6.4      | Cantilever Beam                                                   | 141        |
| 6.5      | Propped Cantilever Beam                                           | 142        |
| 6.6      | Clamped-clamped Beam                                              | 143        |
| 6.6.1    | Effective Lengths                                                 | 145        |
| 6.7      | Energy Method for Buckling of Beams                               | 145        |
| 6.8      | Post-buckling Deflections                                         | 147        |
| 6.9      | Torsional-bending Buckling of a Compressed Beam                   | 150        |
| 6.9.1    | Torsional Buckling                                                | 152        |
| 6.9.2    | General Case                                                      | 152        |
| 6.10     | Example: Split Circular Cross-section Column<br>under Compression | 153        |
| 6.11     | Dynamic Stability of an Airfoil                                   | 154        |
|          | Further Reading                                                   | 155        |
|          | Exercises                                                         | 156        |
| <b>7</b> | <b>Failure Theories</b>                                           | <b>159</b> |
| 7.1      | Brittle Failure                                                   | 159        |
| 7.1.1    | Maximum Principal Stress Theory                                   | 159        |
| 7.1.2    | Mohr-Coulomb Theory                                               | 160        |
| 7.1.3    | Maximum Principal Strain Theory                                   | 160        |
| 7.2      | Failure Theories for Composites                                   | 161        |
| 7.2.1    | Maximum Stress Theory for Composites                              | 162        |
| 7.2.2    | Maximum Strain Theory for Composites                              | 162        |
| 7.2.3    | Tsai-Hill Theory                                                  | 162        |
| 7.3      | Ductile Failure                                                   | 163        |
| 7.3.1    | Tresca Theory                                                     | 164        |
| 7.3.2    | Von Mises Theory                                                  | 164        |
| 7.4      | Fatigue                                                           | 166        |
| 7.4.1    | Palmgren-Minor Rule of Cumulative Damage                          | 167        |
| 7.4.2    | Goodman Diagram                                                   | 167        |
| 7.5      | Creep                                                             | 168        |
| 7.6      | Stress Concentration Factor                                       | 168        |

|       |                                               |     |
|-------|-----------------------------------------------|-----|
| 7.7   | Fracture Mechanics                            | 169 |
| 7.7.1 | Mode I                                        | 170 |
| 7.7.2 | Mode II                                       | 171 |
| 7.7.3 | Mode III                                      | 172 |
| 7.7.4 | Crack Propagation                             | 172 |
| 7.7.5 | Example: Energy Method for a Split Cantilever | 175 |
| 7.7.6 | Ductility at the Crack Tip                    | 175 |
| 7.8   | Fatigue Crack Growth                          | 175 |
|       | Further Reading                               | 176 |
|       | Exercises                                     | 176 |
|       | Index                                         | 179 |