Contents

Preface ..... *v*

Chapter 1 The Real Number System **1**

1-1 Introduction 1
1-2 Preliminaries on Sets 2
1-3 Field Axioms 6
Problem Set 1-3 ..... *13*
1-4 Order Axioms 14
Problem Set 1-4 ..... *19*
1-5 Bounded Subsets 21
Problem Set 1-5 ..... *24*
1-6 Completeness Axiom 26
Problem Set 1-6 ..... *30*
1-7 Intervals 31
Problem Set 1-7 ..... *34*
1-8 System of Integers in $\mathbb{R}$ 35
Problem Set 1-8 ..... *37*
1-9 System of Rational Numbers in $\mathbb{R}$ 39
Problem Set 1-9 ..... *41*
1-10 System $\mathbb{C}$ of Complex Numbers 42
Problem Set 1-10 ..... *45*
1-11 Finite Sums and Products 45
Problem Set 1-11 ..... *52*
Appendix 1 (Completeness and Everywhere Zero Derivative) *53*

Chapter 2 Continuous Functions **56**

2-1 Elementary Properties of Continuous Functions 56
Problem Set 2-1 ..... *63*
2-2 Inverses and Compositions 64
Problem Set 2-2 ..... *66*
2-3 Boundedness Property 68
Problem Set 2-3 ..... *72*

2-4 Intermediate Value Property 73
Problem Set 2-4 ..... *76*
2-5 Uniform Continuity 78
Problem Set 2-5 ..... *82*

Chapter 3 Real and Complex Sequences 84

3-1 Sequences and Their Limits 84
Problem Set 3-1 ..... *90*
3-2 Elementary Results on Convergence and Boundedness 92
Problem Set 3-2 ..... *97*
3-3 Monotone Sequences 98
Problem Set 3-3 ..... *105*
3-4 Subsequences, lim sup and lim inf 106
Problem Set 3-4 ..... *111*
3-5 Cauchy Criterion 112
Problem Set 3-5 ..... *116*

Chapter 4 Real and Complex Series 117

4-1 Convergence of Infinite Series 117
Problem Set 4-1 ..... *123*
4-2 Cauchy Criterion and Absolute Convergence 124
Problem Set 4-2 ..... *127*
4-3 Ratio and Root Tests 128
Problem Set 4-3 ..... *132*
4-4 Abel's and Dirichlet's Tests 133
Problem Set 4-4 ..... *135*
4-5 Infinite Limits of Real Sequences 136
Problem Set 4-5 ..... *140*
4-6 Multiplication of Series 141
Problem Set 4-6 ..... *143*

Chapter 5 Sequences and Series of Functions 144

5-1 Convergence of a Sequence of Functions 144
Problem Set 5-1 ..... *151*
5-2 Series of Functions and Power Series 153
Problem Set 5-2 ..... *159*
5-3 Special Functions 160
Problem Set 5-3 ..... *168*

Chapter 6 Uniform Approximation 170

6-1 Heine-Borel Theorem 170
Problem Set 6-1 ..... *173*

6-2 Polynomials and Step Functions 174
Problem Set 6-2 ..... *180*
6-3 Stone-Weierstrass Theorem 181
Problem Set 6-3 ..... *188*

Chapter 7 Differentiation 191

7-1 Preliminaries on Limit Points and Limits 191
Problem Set 7-1 ..... *198*
7-2 Definition of Derivative and Elementary Properties 199
Problem Set 7-2 ..... *206*
7-3 Powers, Trigonometric Functions and Their Inverses 207
Problem Set 7-3 ..... *213*
7-4 Exponential Function and its Inverse 213
Problem Set 7-4 ..... *220*
7-5 Lagrange Mean Value Theorem 221
Problem Set 7-5 ..... *229*
7-6 Taylor's Theorem 232
Problem Set 7-6 ..... *239*

Chapter 8 More About Differentiation 241

8-1 Applications of Taylor's Theorem 241
Problem Set 8-1 ..... *245*
8-2 Cauchy Mean Value Theorem 246
Problem Set 8-2 ..... *252*
8-3 Differentiability of Uniform Limits 253
Problem Set 8-3 ..... *256*
8-4 Continuous Nowhere Differentiable Function 257
Problem Set 8-4 ..... *262*
Appendix 1 (Irrationality of π) *262*
Appendix 2 (Pre-Calculus Functions and $\mathbb{R}$) *263*

Chapter 9 The Riemann Integral 266

9-1 Introduction 266
Problem Set 9-1 ..... *270*
9-2 Definition of the Riemann Integral 272
Problem Set 9-2 ..... *278*
9-3 Riemann Integrable Functions 279
Problem Set 9-3 ..... *284*
9-4 Properties of the Riemann Integral 284
Problem Set 9-4 ..... *291*
9-5 Fundamental Theorem of Calculus 293
Problem Set 9-5 ..... *298*

Chapter 10 More About the Riemann Integral 301

10-1 Integral as a Limit 301

Problem Set 10-1 ..... *306*

10-2 Improper Integrals 306

Problem Set 10-2 ..... *311*

10-3 Mean Value Theorems and Applications 312

Problem Set 10-3 ..... *317*

10-4 $\int_0^\infty \exp(-x^2)dx = \sqrt{\pi}/2$ 318

Problem Set 10-4 ..... *319*

Chapter 11 The Stieltjes Integral 320

11-1 Introduction 320

Problem Set 11-1 ..... *322*

11-2 Definition of the Stieltjes Integral 323

Problem Set 11-2 ..... *328*

11-3 Stieltjes Integrable Functions 329

Problem Set 11-3 ..... *333*

11-4 Properties of the Stieltjes Integral 333

Problem Set 11-4 ..... *338*

HINTS 339

Problem Set 1-3 339
Problem Set 1-4 339
Problem Set 1-5 339
Problem Set 1-6 340
Problem Set 1-7 340
Problem Set 1-8 340
Problem Set 1-9 341
Problem Set 1-10 341
Problem Set 1-11 341
Problem Set 2-1 341
Problem Set 2-2 342
Problem Set 2-3 342
Problem Set 2-4 342
Problem Set 2-5 343
Problem Set 3-1 343
Problem Set 3-2 344
Problem Set 3-3 344
Problem Set 3-4 344
Problem Set 3-5 345
Problem Set 4-1 345
Problem Set 4-2 345
Problem Set 4-3 345
Problem Set 4-4 346

Problem Set 4-5 346
Problem Set 4-6 346
Problem Set 5-1 346
Problem Set 5-2 347
Problem Set 5-3 347
Problem Set 6-1 348
Problem Set 6-2 348
Problem Set 6-3 348
Problem Set 7-1 349
Problem Set 7-2 349
Problem Set 7-3 349
Problem Set 7-4 349
Problem Set 7-5 350
Problem Set 7-6 350
Problem Set 8-1 351
Problem Set 8-2 351
Problem Set 8-3 351
Problem Set 9-1 351
Problem Set 9-2 351
Problem Set 9-3 352
Problem Set 9-4 352
Problem Set 9-5 352
Problem Set 10-1 353
Problem Set 10-2 353
Problem Set 10-3 353
Problem Set 10-4 354
Problem Set 11-1 354
Problem Set 11-2 354
Problem Set 11-3 355
Problem Set 11-4 355

References ***356***
Index ***357***