

CONTENTS

Preface to the SI Edition xii

Preface xiii

Acknowledgments xv

Notation xvi

1 Introduction 1

Chapter Objectives 1

Prologue 1

1.1 Brief History 3

1.2 Introduction to Matrix Notation 4

1.3 Role of the Computer 6

1.4 General Steps of the Finite Element Method 7

1.5 Applications of the Finite Element Method 15

1.6 Advantages of the Finite Element Method 21

1.7 Computer Programs for the Finite Element Method 25

Reference 27

Problems 30

2 Introduction to the Stiffness (Displacement) Method 31

Chapter Objectives 31

Introduction 31

2.1 Definition of the Stiffness Matrix 32

2.2 Derivation of the Stiffness Matrix
for a Spring Element 32

2.3 Example of a Spring Assemblage 36

2.4 Assembling the Total Stiffness Matrix by Superposition
(Direct Stiffness Method) 38

2.5 Boundary Conditions 40

2.6 Potential Energy Approach to Derive Spring Element Equations 55

Summary Equations 65

References 66

Problems 66

3 Development of Truss Equations 72

- Chapter Objectives 72
- Introduction 72
- 3.1** Derivation of the Stiffness Matrix for a Bar Element in Local Coordinates 73
- 3.2** Selecting a Displacement Function in Step 2 of the Derivation of Stiffness Matrix for the One-Dimensional Bar Element 78
- 3.3** Transformation of Vectors in Two Dimensions 82
- 3.4** Global Stiffness Matrix for Bar Arbitrarily Oriented in the Plane 84
- 3.5** Computation of Stress for a Bar in the $x - y$ Plane 89
- 3.6** Solution of a Plane Truss 91
- 3.7** Transformation Matrix and Stiffness Matrix for a Bar in Three-Dimensional Space 100
- 3.8** Use of Symmetry in Structures 109
- 3.9** Inclined, or Skewed, Supports 112
- 3.10** Potential Energy Approach to Derive Bar Element Equations 121
- 3.11** Comparison of Finite Element Solution to Exact Solution for Bar 132
- 3.12** Galerkin's Residual Method and Its Use to Derive the One-Dimensional Bar Element Equations 136
- 3.13** Other Residual Methods and Their Application to a One-Dimensional Bar Problem 139
- 3.14** Flowchart for Solution of Three-Dimensional Truss Problems 143
- 3.15** Computer Program Assisted Step-by-Step Solution for Truss Problem 144
- Summary Equations 146
- References 147
- Problems 147

4 Development of Beam Equations 169

- Chapter Objectives 169
- Introduction 169
- 4.1** Beam Stiffness 170
- 4.2** Example of Assemblage of Beam Stiffness Matrices 180
- 4.3** Examples of Beam Analysis Using the Direct Stiffness Method 182
- 4.4** Distributed Loading 195

- 4.5 Comparison of the Finite Element Solution to the Exact Solution for a Beam 208
- 4.6 Beam Element with Nodal Hinge 214
- 4.7 Potential Energy Approach to Derive Beam Element Equations 222
- 4.8 Galerkin's Method for Deriving Beam Element Equations 225
- Summary Equations 227
- References 228
- Problems 229

5 Frame and Grid Equations 239

- Chapter Objectives 239
- Introduction 239
- 5.1 Two-Dimensional Arbitrarily Oriented Beam Element 239
- 5.2 Rigid Plane Frame Examples 243
- 5.3 Inclined or Skewed Supports—Frame Element 261
- 5.4 Grid Equations 262
- 5.5 Beam Element Arbitrarily Oriented in Space 280
- 5.6 Concept of Substructure Analysis 295
- Summary Equations 300
- References 302
- Problems 303

6 Development of the Plane Stress and Plane Strain Stiffness Equations 337

- Chapter Objectives 337
- Introduction 337
- 6.1 Basic Concepts of Plane Stress and Plane Strain 338
- 6.2 Derivation of the Constant-Strain Triangular Element Stiffness Matrix and Equations 342
- 6.3 Treatment of Body and Surface Forces 357
- 6.4 Explicit Expression for the Constant-Strain Triangle Stiffness Matrix 362
- 6.5 Finite Element Solution of a Plane Stress Problem 363
- 6.6 Rectangular Plane Element (Bilinear Rectangle, Q4) 374
- Summary Equations 379
- References 384
- Problems 384

7 Practical Considerations in Modeling; Interpreting Results; and Examples of Plane Stress/Strain Analysis 391

Chapter Objectives 391

Introduction 391

- 7.1 Finite Element Modeling 392
 - 7.2 Equilibrium and Compatibility of Finite Element Results 405
 - 7.3 Convergence of Solution and Mesh Refinement 408
 - 7.4 Interpretation of Stresses 411
 - 7.5 Flowchart for the Solution of Plane Stress/Strain Problems 413
 - 7.6 Computer Program-Assisted Step-by-Step Solution, Other Models, and Results for Plane Stress/Strain Problems 414
- References 420
- Problems 421

8 Development of the Linear-Strain Triangle Equations 437

Chapter Objectives 437

Introduction 437

- 8.1 Derivation of the Linear-Strain Triangular Element Stiffness Matrix and Equations 437
 - 8.2 Example LST Stiffness Determination 442
 - 8.3 Comparison of Elements 444
- Summary Equations 447
- References 448
- Problems 448

9 Axisymmetric Elements 451

Chapter Objectives 451

Introduction 451

- 9.1 Derivation of the Stiffness Matrix 451
 - 9.2 Solution of an Axisymmetric Pressure Vessel 462
 - 9.3 Applications of Axisymmetric Elements 468
- Summary Equations 473
- References 475
- Problems 475

10 Isoparametric Formulation 486

Chapter Objectives 486

Introduction 486

- 10.1 Isoparametric Formulation of the Bar Element Stiffness Matrix 487
- 10.2 Isoparametric Formulation of the Plane Quadrilateral (Q4) Element Stiffness Matrix 492
- 10.3 Newton-Cotes and Gaussian Quadrature 503
- 10.4 Evaluation of the Stiffness Matrix and Stress Matrix by Gaussian Quadrature 509
- 10.5 Higher-Order Shape Functions (Including Q6, Q8, Q9, and Q12 Elements) 515
 - Summary Equations 526
 - References 530
 - Problems 530
- 11 Three-Dimensional Stress Analysis 536**
 - Chapter Objectives 536
 - Introduction 536
 - 11.1 Three-Dimensional Stress and Strain 537
 - 11.2 Tetrahedral Element 539
 - 11.3 Isoparametric Formulation and Hexahedral Element 547
 - Summary Equations 555
 - References 558
 - Problems 558
- 12 Plate Bending Element 572**
 - Chapter Objectives 572
 - Introduction 572
 - 12.1 Basic Concepts of Plate Bending 572
 - 12.2 Derivation of a Plate Bending Element Stiffness Matrix and Equations 577
 - 12.3 Some Plate Element Numerical Comparisons 582
 - 12.4 Computer Solutions for Plate Bending Problems 584
 - Summary Equations 588
 - References 590
 - Problems 591
- 13 Heat Transfer and Mass Transport 599**
 - Chapter Objectives 599
 - Introduction 599
 - 13.1 Derivation of the Basic Differential Equation 601
 - 13.2 Heat Transfer with Convection 604

- 13.3 Typical Units; Thermal Conductivities, K ; and Heat Transfer Coefficients, h 605
- 13.4 One-Dimensional Finite Element Formulation Using a Variational Method 607
- 13.5 Two-Dimensional Finite Element Formulation 626
- 13.6 Line or Point Sources 636
- 13.7 Three-Dimensional Heat Transfer by the Finite Element Method 639
- 13.8 One-Dimensional Heat Transfer with Mass Transport 641
- 13.9 Finite Element Formulation of Heat Transfer with Mass Transport by Galerkin's Method 642
- 13.10 Flowchart and Examples of a Heat Transfer Program 646
 - Summary Equations 651
 - References 654
 - Problems 655

14 Fluid Flow in Porous Media and through Hydraulic Networks; and Electrical Networks and Electrostatics 673

- Chapter Objectives 673
- Introduction 673
- 14.1 Derivation of the Basic Differential Equations 674
- 14.2 One-Dimensional Finite Element Formulation 678
- 14.3 Two-Dimensional Finite Element Formulation 691
- 14.4 Flowchart and Example of a Fluid-Flow Program 696
- 14.5 Electrical Networks 697
- 14.6 Electrostatics 701
 - Summary Equations 715
 - References 719
 - Problems 720

15 Thermal Stress 727

- Chapter Objectives 727
- Introduction 727
- 15.1 Formulation of the Thermal Stress Problem and Examples 727
 - Summary Equations 752
 - Reference 753
 - Problems 754

16 Structural Dynamics and Time-Dependent Heat Transfer 761

Chapter Objectives 761

Introduction 761

- 16.1** Dynamics of a Spring-Mass System 762
 - 16.2** Direct Derivation of the Bar Element Equations 764
 - 16.3** Numerical Integration in Time 768
 - 16.4** Natural Frequencies of a One-Dimensional Bar 780
 - 16.5** Time-Dependent One-Dimensional Bar Analysis 784
 - 16.6** Beam Element Mass Matrices and Natural Frequencies 789
 - 16.7** Truss, Plane Frame, Plane Stress, Plane Strain, Axisymmetric, and Solid Element Mass Matrices 796
 - 16.8** Time-Dependent Heat Transfer 801
 - 16.9** Computer Program Example Solutions for Structural Dynamics 808
- Summary Equations 817
- References 821
- Problems 822

Appendix A Matrix Algebra 827

Appendix B Methods for Solution of Simultaneous Linear Equations 843

Appendix C Equations from Elasticity Theory 865

Appendix D Equivalent Nodal Forces 873

Appendix E Principle of Virtual Work 876

Appendix F Geometric Properties of Structural Steel Wide-Flange Sections (W Shapes) 880

Answers to Selected Problems 908

Index 938