

Contents

Preface	xiii
1 Introduction	1
1.1 Historical Perspective, 1	
1.2 Kinematics, 3	
1.3 Design: Analysis and Synthesis, 4	
1.4 Mechanisms, 4	
1.5 Planar Linkages, 6	
1.6 Visualization, 9	
1.7 Constraint Analysis, 12	
1.8 Constraint Analysis of Spatial Linkages, 18	
1.9 Idle Degrees of Freedom, 22	
1.10 Overconstrained Linkages, 24	
1.11 Uses of the Mobility Criterion, 28	
1.12 Inversion, 28	
1.13 Reference Frames, 29	
1.14 Motion Limits, 30	
1.15 Continuously Rotatable Joints, 31	
1.16 Coupler-Driven Linkages, 35	
1.17 Motion Limits for Slider-Crank Mechanisms, 35	
1.18 Interference, 38	
1.19 Practical Design Considerations, 41	
1.19.1 Revolute Joints, 41	
1.19.2 Prismatic Joints, 42	
1.19.3 Higher Pairs, 44	
1.19.4 Cams versus Linkages, 44	
References, 44	
Problems, 45	
2 Techniques in Geometric Constraint Programming	59
2.1 Introduction, 59	
2.2 Geometric Constraint Programming, 60	
2.3 Constraints and Program Structure, 61	
2.3.1 Required Constraints, 61	
2.3.2 Other Constraint Options, 62	
2.3.3 Annotations, 62	
2.3.4 Use of Drawing Layers, 62	
2.3.5 Limitations of GCP, 63	
2.4 Initial Setup for a GCP Session, 64	
2.4.1 Effect of Typical Constraints, 64	
2.4.2 Unintended Constraints, 66	
2.4.3 Layers, Line Type, and Line Color, 66	
2.5 Drawing a Basic Linkage Using GCP, 66	
2.5.1 Drawing a Four-Bar Linkage Using GCP, 66	
2.5.2 Including Ground Pivots and Bushings, 68	
2.5.3 Drawing a Slider-Crank Linkage, 68	
2.6 Troubleshooting Graphical Programs Developed Using GCP, 79	

- References, 80
- Problems, 81
- Appendix 2A Drawing Slider Lines, Pin Bushings, and Ground Pivots, 85
 - 2A.1 Slider Lines, 85
 - 2A.2 Pin Bushings and Ground Pivots, 87
- Appendix 2B Useful Constructions When Equation Constraints Are Not Available, 88
 - 2B.1 Constrain Two Angles to Be Integral Multiples of Another Angle, 89
 - 2B.2 Constrain a Line to Be Half the Length of Another Line, 89
 - 2B.3 Construction for Scaling, 90
 - 2B.4 Construction for Square Ratio v^2/r , 91
 - 2B.5 Construction for Function $x = yz/r$, 91

3 Planar Linkage Design

93

- 3.1 Introduction, 93
- 3.2 Two-Position Double-Rocker Design, 96
 - 3.2.1 Graphical Solution Procedure, 97
 - 3.2.2 Solution Using Geometric Constraint Programming, 97
 - 3.2.3 Numerical Solution Procedure, 99
- 3.3 Synthesis of Crank-Rocker Linkages for Specified Rocker Amplitude, 100
 - 3.3.1 The Rocker-Amplitude Problem: Graphical Approach, 100
 - 3.3.2 Alternative Graphical Design Procedure Based on Specification of A^*B^* , 105
 - 3.3.3 Using GCP to Design Crank-Rocker and Crank-Shaper Mechanisms, 107
- 3.4 Motion Generation, 114
 - 3.4.1 Introduction, 114
 - 3.4.2 Two Positions, 114
 - 3.4.3 Three Positions with Selected Moving Pivots, 116
 - 3.4.4 Synthesis of a Crank with Chosen Fixed Pivots, 117
 - 3.4.5 Design of Slider-Cranks and Elliptic-Trammels, 118
 - 3.4.6 Change of Branch, 118
 - 3.4.7 Using GCP for Rigid-Body Guidance, 124
- 3.5 Path Synthesis, 133
 - 3.5.1 Design of Six-Bar Linkages Using Coupler Curves, 133
 - 3.5.2 Motion Generation for Parallel Motion Using Coupler Curves, 138
 - 3.5.3 Cognate Linkages, 141
 - 3.5.4 Using GCP for Path Synthesis, 144
- References, 148
- Problems, 150

4 Graphical Position, Velocity, and Acceleration Analysis for Mechanisms with Revolute Joints or Fixed Slides

169

- 4.1 Introduction, 169
- 4.2 Graphical Position Analysis, 170
- 4.3 Planar Velocity Polygons, 171
- 4.4 Graphical Acceleration Analysis, 173
- 4.5 Graphical Analysis of a Four-Bar Mechanism, 175
- 4.6 Graphical Analysis of a Slider-Crank Mechanism, 183
- 4.7 Velocity Image Theorem, 186
- 4.8 Acceleration Image Theorem, 189
- 4.9 Solution by Geometric Constraint Programming, 194
 - 4.9.1 Introduction, 194
 - 4.9.2 Scaling Property of Velocity Polygons, 195
 - 4.9.3 Using GCP to Analyze Linkages That Cannot Be Analyzed by Classical Means for Velocities, 199
- References, 205
- Problems, 205

5 Linkages with Rolling and Sliding Contacts, and Joints on Moving Sliders 221

- 5.1 Introduction, 221
- 5.2 Reference Frames, 222
- 5.3 General Velocity and Acceleration Equations, 223
 - 5.3.1 Velocity Equations, 223
 - 5.3.2 Acceleration Equations, 225
 - 5.3.3 Chain Rule for Positions, Velocities, and Accelerations, 226
- 5.4 Special Cases for the Velocity and Acceleration Equations, 228
 - 5.4.1 Two Points Fixed in a Moving Body, 228
 - 5.4.2 Two Points Are Instantaneously Coincident, 229
 - 5.4.3 Two Points Are Instantaneously Coincident and in Rolling Contact, 230
- 5.5 Linkages with Rotating Sliding Joints, 230
- 5.6 Rolling Contact, 235
 - 5.6.1 Basic Kinematic Relationships for Rolling Contact, 235
 - 5.6.2 Modeling Rolling Contact Using a Virtual Linkage, 241
- 5.7 Cam Contact, 243
 - 5.7.1 Direct Approach to the Analysis of Cam Contact, 243
 - 5.7.2 Analysis of Cam Contact Using Equivalent Linkages, 246
- 5.8 General Coincident Points, 250
 - 5.8.1 Velocity Analyses Involving General Coincident Points, 250
 - 5.8.2 Acceleration Analyses Involving General Coincident Points, 251
- 5.9 Solution by Geometric Constraint Programming, 257

Problems, 263

6 Instant Centers of Velocity 279

- 6.1 Introduction, 279
- 6.2 Definition, 280
- 6.3 Existence Proof, 280
- 6.4 Location of an Instant Center from the Directions of Two Velocities, 281
- 6.5 Instant Center at a Revolute Joint, 282
- 6.6 Instant Center of a Curved Slider, 282
- 6.7 Instant Center of a Prismatic Joint, 282
- 6.8 Instant Center of a Rolling Contact Pair, 282
- 6.9 Instant Center of a General Cam-Pair Contact, 282
- 6.10 Centrodes, 283
- 6.11 The Kennedy-Aronhold Theorem, 285
- 6.12 Circle Diagram as a Strategy for Finding Instant Centers, 287
- 6.13 Using Instant Centers to Find Velocities: The Rotating-Radius Method, 287
- 6.14 Finding Instant Centers Using Geometric Constraint Programming, 295

References, 300

Problems, 300

7 Computational Analysis of Linkages 315

- 7.1 Introduction, 315
- 7.2 Position, Velocity, and Acceleration Representations, 316
 - 7.2.1 Position Representation, 316
 - 7.2.2 Velocity Representation, 316
 - 7.2.3 Acceleration Representation, 317
 - 7.2.4 Special Cases, 318
 - 7.2.5 Mechanisms to Be Considered, 319
- 7.3 Analytical Closure Equations for Four-Bar Linkages, 319
 - 7.3.1 Solution of Closure Equations for Four-Bar Linkages When Link 2 Is the Driver, 319
 - 7.3.2 Analysis When the Coupler (Link 3) Is the Driving Link, 322

7.3.3	Velocity Equations for Four-Bar Linkages, 322
7.3.4	Acceleration Equations for Four-Bar Linkages, 323
7.4	Analytical Equations for a Rigid Body after the Kinematic Properties of Two Points Are Known, 326
7.5	Analytical Equations for Slider-Crank Mechanisms, 329
7.5.1	Solution to Position Equations When θ_2 Is Input, 330
7.5.2	Solution to Position Equations When r_1 Is Input, 332
7.5.3	Solution to Position Equations When θ_3 Is Input, 333
7.5.4	Velocity Equations for Slider-Crank Mechanism, 334
7.5.5	Acceleration Equations for Slider-Crank Mechanism, 334
7.6	Other Four-Bar Mechanisms with Revolute and Prismatic Joints, 338
7.6.1	Slider-Crank Inversion, 339
7.6.2	A RPRP Mechanism, 339
7.6.3	A RRPP Mechanism, 340
7.6.4	Elliptic-Trammel Mechanism, 340
7.6.5	Oldham Mechanism, 341
7.7	Closure or Loop Equation Approach for Compound Mechanisms, 341
7.7.1	Handling Points Not on the Vector Loops, 344
7.7.2	Solving the Position Equations, 345
7.8	Closure Equations for Mechanisms with Higher Pairs, 347
7.9	Notational Differences: Vectors and Complex Numbers, 352
	Problems, 354

8 Special Mechanisms

361

8.1	Special Planar Mechanisms, 361
8.1.1	Introduction, 361
8.1.2	Straight-Line and Circle Mechanisms, 362
8.1.3	Pantographs, 368
8.2	Spherical Mechanisms, 374
8.2.1	Introduction, 374
8.2.2	Gimbals, 376
8.2.3	Universal Joints, 377
8.3	Constant-Velocity Couplings, 381
8.3.1	Geometric Requirements of Constant-Velocity Couplings, 381
8.3.2	Practical Constant-Velocity Couplings, 381
8.4	Automotive Steering and Suspension Mechanisms, 382
8.4.1	Introduction, 382
8.4.2	Steering Mechanisms, 382
8.4.3	Suspension Mechanisms, 386
8.5	Indexing Mechanisms, 387
8.5.1	Geneva Mechanisms, 387
	References, 392
	Problems, 392

9 Computational Analysis of Spatial Linkages

395

9.1	Spatial Mechanisms, 395
9.1.1	Introduction, 395
9.1.2	Velocity and Acceleration Relationships, 396
9.2	Robotic Mechanisms, 401
9.3	Direct Position Kinematics of Serial Chains, 403
9.3.1	Introduction, 403
9.3.2	Concatenation of Transformations, 404
9.3.3	Homogeneous Transformations, 407
9.4	Inverse Position Kinematics, 410

- 9.5 Rate Kinematics, 410
 - 9.5.1 Introduction, 410
 - 9.5.2 Direct Rate Kinematics, 410
 - 9.5.3 Inverse Rate Kinematics, 414
- 9.6 Closed-Loop Linkages, 416
- 9.7 Lower-Pair Joints, 418
- 9.8 Motion Platforms, 421
 - 9.8.1 Mechanisms Actuated in Parallel, 421
 - 9.8.2 The Stewart-Gough Platform, 422
 - 9.8.3 The 3-2-1 Platform, 423

References, 423

Problems, 423

10 Profile Cam Design

431

- 10.1 Introduction, 431
- 10.2 Cam-Follower Systems, 432
- 10.3 Synthesis of Motion Programs, 434
- 10.4 Analysis of Different Types of Follower-Displacement Functions, 436
 - 10.4.1 Uniform Motion, 437
 - 10.4.2 Parabolic Motion, 438
 - 10.4.3 Harmonic Follower-Displacement Programs, 443
 - 10.4.4 Cycloidal Follower-Displacement Programs, 444
 - 10.4.5 General Polynomial Follower-Displacement Programs, 445
- 10.5 Determining the Cam Profile, 448
 - 10.5.1 Graphical Cam Profile Layout, 450
 - 10.5.2 Analytical Determination of Cam Profile, 460

References, 482

Problems, 482

11 Spur Gears

489

- 11.1 Introduction, 489
- 11.2 Spur Gears, 490
- 11.3 Condition for Constant-Velocity Ratio, 491
- 11.4 Involutives, 492
- 11.5 Gear Terminology and Standards, 494
 - 11.5.1 Terminology, 494
 - 11.5.2 Standards, 496
- 11.6 Contact Ratio, 497
- 11.7 Involutometry, 501
- 11.8 Internal Gears, 504
- 11.9 Gear Manufacturing, 505
- 11.10 Interference and Undercutting, 508
- 11.11 Nonstandard Gearing, 510
- 11.12 Cartesian Coordinates of an Involute Tooth Generated with a Rack, 514
 - 11.12.1 Coordinate Systems, 514
 - 11.12.2 Gear Equations, 516

References, 520

Problems, 520

12 Helical, Bevel, and Worm Gears

523

- 12.1 Helical Gears, 523
 - 12.1.1 Helical Gear Terminology, 523

- 12.1.2 Helical Gear Manufacturing, 526
- 12.1.3 Minimum Tooth Number to Avoid Undercutting, 526
- 12.1.4 Helical Gears with Parallel Shafts, 527
- 12.1.5 Crossed Helical Gears, 533
- 12.2 Worm Gears, 536
 - 12.2.1 Worm Gear Nomenclature, 537
- 12.3 Involute Bevel Gears, 540
 - 12.3.1 Tredgold's Approximation for Bevel Gears, 541
 - 12.3.2 Additional Nomenclature for Bevel Gears, 542
 - 12.3.3 Crown Bevel Gears and Face Gears, 543
 - 12.3.4 Miter Gears, 544
 - 12.3.5 Angular Bevel Gears, 544
 - 12.3.6 Zerol Bevel Gears, 544
 - 12.3.7 Spiral Bevel Gears, 545
 - 12.3.8 Hypoid Gears, 546
- References, 547
- Problems, 547

13 Gear Trains

549

- 13.1 General Gear Trains, 549
- 13.2 Direction of Rotation, 549
- 13.3 Simple Gear Trains, 550
 - 13.3.1 Simple Reversing Mechanism, 551
- 13.4 Compound Gear Trains, 552
 - 13.4.1 Concentric Gear Trains, 555
- 13.5 Planetary Gear Trains, 558
 - 13.5.1 Planetary Gear Nomenclature, 558
 - 13.5.2 Analysis of Planetary Gear Trains Using Equations, 559
 - 13.5.3 Analysis of Planetary Gear Trains Using Tabular Method, 567
- 13.6 Harmonic Drive Speed Reducers, 570
- References, 572
- Problems, 572

14 Static Force Analysis of Mechanisms

579

- 14.1 Introduction, 579
- 14.2 Forces, Moments, and Couples, 580
- 14.3 Static Equilibrium, 581
- 14.4 Free-Body Diagrams, 582
- 14.5 Solution of Static Equilibrium Problems, 585
- 14.6 Transmission Angle in a Four-Bar Linkage, 587
- 14.7 Friction Considerations, 590
 - 14.7.1 Friction in Cam Contact, 591
 - 14.7.2 Friction in Slider Joints, 591
 - 14.7.3 Friction in Revolute Joints, 593
- 14.8 In-Plane and Out-of-Plane Force Systems, 597
- 14.9 Conservation of Energy and Power, 601
- 14.10 Virtual Work, 605
- 14.11 Gear Loads, 607
 - 14.11.1 Spur Gears, 607
 - 14.11.2 Helical Gears, 609
 - 14.11.3 Worm Gears, 611
 - 14.11.4 Straight Bevel Gears, 612
- Problems, 613

15	Dynamic Force Analysis of Mechanisms	623
15.1	Introduction, 623	
15.2	Problems Solvable Using Particle Kinetics, 625	
15.2.1	Dynamic Equilibrium of Systems of Particles, 625	
15.2.2	Conservation of Energy, 630	
15.2.3	Conservation of Momentum, 630	
15.3	Dynamic Equilibrium of Systems of Rigid Bodies, 633	
15.4	Flywheels, 639	
	Problems, 641	
16	Static and Dynamic Balancing	645
16.1	Introduction, 645	
16.2	Single-Plane (Static) Balancing, 646	
16.3	Multi-Plane (Dynamic) Balancing, 649	
16.4	Balancing Reciprocating Masses, 654	
16.4.1	Lumped Mass Distribution, 655	
16.4.2	Balancing a Slider-Crank Mechanism, 658	
16.5	Expressions for Inertial Forces, 661	
16.6	Balancing Multi-Cylinder Machines, 663	
16.6.1	Balancing a Three-Cylinder In-Line Engine, 667	
16.6.2	Balancing an Eight-Cylinder V Engine, 669	
16.7	Static Balancing of Mechanisms, 671	
16.7.1	Gravity Balancing of Planar Mechanisms: Examples, 672	
16.7.2	Gravity-Balancing Orthosis, 675	
16.8	Reactionless Mechanisms, 675	
	References, 676	
	Problems, 676	
17	Integration of Computer Controlled Actuators	685
17.1	Introduction, 685	
17.2	Computer Control of the Linkage Motion, 686	
17.3	The Basics of Feedback Control, 687	
17.4	Actuator Selection and Types, 688	
17.4.1	Electric Actuation, 689	
17.4.2	Hydraulic Actuation, 692	
17.4.3	Pneumatic Actuation, 693	
17.5	Hands-On Machine-Design Laboratory, 694	
17.5.1	Examples of Class Projects, 695	
	References, 696	
	Problems, 696	
	Index	699