Contents

Preface *page* xiii

1 Introduction: the need for a quantum theory 1
1.1 Introducing quantum mechanics 1

2 Experimental foundations of quantum theory 5
2.1 Black-body radiation 5
2.1.1 Kirchhoff laws 6
2.1.2 Electromagnetic field in a hollow cavity 7
2.1.3 Stefan and displacement laws 9
2.1.4 Planck model 13
2.1.5 Contributions of Einstein 17
2.1.6 Dynamic equilibrium of the radiation field 19
2.2 Photoelectric effect 19
2.2.1 Classical model 21
2.2.2 Quantum theory of the effect 23
2.3 Compton effect 25
2.3.1 Thomson scattering 29
2.4 Particle-like behaviour and the Heisenberg picture 30
2.4.1 Atomic spectra and the Bohr hypotheses 30
2.5 Corpuscular character: the experiment of Franck and Hertz 34
2.6 Wave-like behaviour and the Bragg experiment 35
2.6.1 Connection between the wave picture and the discrete-level picture 35
2.7 Experiment of Davisson and Germer 39
2.8 Interference phenomena among material particles 41
Appendix 2.A Classical electrodynamics and the Planck formula 46

3 Waves and particles 51
3.1 Waves: d'Alembert equation 51
3.2 Particles: Hamiltonian equations 58
3.2.1 Poisson brackets among velocity components for a charged particle 62
3.3 Homogeneous linear differential operators and equations of motion 64
3.4 Symmetries and conservation laws 65

3.4.1 Homomorphism between $SU(2)$ and $SO(3)$ 67
3.5 Motivations for studying harmonic oscillators 72
3.6 Complex coordinates for harmonic oscillators 74
3.7 Canonical transformations 75
3.8 Time-dependent Hamiltonian formalism 76
3.9 Hamilton–Jacobi equation 78
3.10 Motion of surfaces 81
Appendix 3.A Space–time picture 83
3.A.1 Inertial frames and comparison dynamics 84
3.A.2 Lagrangian descriptions of second-order differential equations 85
3.A.3 Symmetries and constants of motion 88
3.A.4 Symmetries and constants of motion in the Hamiltonian formalism 91
3.A.5 Equivalent reference frames 92

4 Schrödinger picture, Heisenberg picture and probabilistic aspects 94
4.1 From classical to wave mechanics 94
4.1.1 Properties of the Schrödinger equation 96
4.1.2 Physical interpretation of the wave function 100
4.1.3 Mean values 103
4.1.4 Eigenstates and eigenvalues 106
4.2 Probability distributions associated with vectors in Hilbert spaces 106
4.3 Uncertainty relations for position and momentum 109
4.4 Transformation properties of wave functions 111
4.4.1 Direct approach to the transformation properties of the Schrödinger equation 113
4.4.2 Width of the wave packet 114
4.5 Heisenberg picture 115
4.6 States in the Heisenberg picture 119
4.7 'Conclusions': relevant mathematical structures 120

5 Integrating the equations of motion 122
5.1 Green kernel of the Schrödinger equation 122
5.1.1 Discrete version of the Green kernel by using a fundamental set of solutions 125
5.1.2 General considerations on how we use solutions of the evolution equation 127
5.2 Integrating the equations of motion in the Heisenberg picture: harmonic oscillator 129

6 Elementary applications: one-dimensional problems 131
6.1 Boundary conditions 131
6.1.1 Particle confined by a potential 132
6.1.2 A closer look at improper eigenfunctions 134

6.2 Reflection and transmission 135
6.3 Step-like potential 139
6.3.1 Tunnelling effect 142
6.4 One-dimensional harmonic oscillator 143
6.4.1 Hermite polynomials 146
6.5 Problems 147
Appendix 6.A Wave-packet behaviour at large time values 148

7 Elementary applications: multi-dimensional problems 151
7.1 The Schrödinger equation in a central potential 151
7.1.1 Use of symmetries and geometrical interpretation 158
7.1.2 Angular momentum operators and spherical harmonics 159
7.1.3 Angular momentum eigenvalues: algebraic treatment 162
7.1.4 Radial part of the eigenvalue problem in a central potential 163
7.2 Hydrogen atom 165
7.2.1 Runge–Lenz vector 168
7.3 *s*-Wave bound states in the square-well potential 170
7.4 Isotropic harmonic oscillator in three dimensions 172
7.5 Multi-dimensional harmonic oscillator: algebraic treatment 174
7.5.1 An example: two-dimensional isotropic harmonic oscillator 175
7.6 Problems 177

8 Coherent states and related formalism 180
8.1 General considerations on harmonic oscillators and coherent states 180
8.2 Quantum harmonic oscillator: a brief summary 182
8.3 Operators in the number operator basis 185
8.4 Representation of states on phase space, the Bargmann–Fock representation 186
8.4.1 The Weyl displacement operator 188
8.5 Basic operators in the coherent states' basis 190
8.6 Uncertainty relations 191
8.7 Ehrenfest picture 192
8.8 Problems 194

9 Introduction to spin 195
9.1 Stern–Gerlach experiment and electron spin 195
9.2 Wave functions with spin 199
9.3 Addition of orbital and spin angular momenta 201
9.4 The Pauli equation 203
9.5 Solutions of the Pauli equation 205
9.5.1 Another simple application of the Pauli equation 207
9.6 Landau levels 209
9.7 Spin–orbit interaction: Thomas precession 210
9.8 Problems 212

10 Symmetries in quantum mechanics 214
10.1 Meaning of symmetries 214
10.1.1 Transformations that preserve the description 216
10.2 Transformations of frames and corresponding quantum symmetries 222
10.2.1 Rototranslations 222
10.3 Galilei transformations 226
10.4 Time translation 229
10.5 Spatial reflection 230
10.6 Time reversal 232
10.7 Problems 232

11 Approximation methods 234

11A Perturbation theory 235
11A.1 Approximation of eigenvalues and eigenvectors 235
11A.2 Hellmann–Feynman theorem 239
11A.3 Virial theorem 241
11A.4 Anharmonic oscillator 245
11A.5 Secular equation for problems with degeneracy 248
11A.6 Stark effect 249
11A.7 Zeeman effect 251
11A.8 Anomalous Zeeman effect 254
11A.9 Relativistic corrections (α^2) to the hydrogen atom 256
11A.10 Variational method 258
11A.11 Time-dependent formalism 259
11A.12 Harmonic perturbations 261
11A.13 Fermi golden rule 263
11A.14 Towards limiting cases of time-dependent theory 263
11A.15 Adiabatic switch on and off of the perturbation 266
11A.16 Perturbation suddenly switched on 266
11A.17 Two-level system 267
11A.18 The quantum K^0–$\overline{K^0}$ system 269
11A.19 The quantum system of three active neutrinos 271

11B Jeffreys–Wentzel–Kramers–Brillouin method 274
11B.1 The JWKB method 274
11B.2 Potential barrier 277
11B.3 Energy levels in a potential well 278
11B.4 α-decay 279

11C Scattering theory 282
11C.1 Aims and problems of quantum scattering theory 282
11C.2 Time-dependent scattering 282
11C.3 An example: classical scattering 284

11C.4 Time-independent scattering 287
11C.4.1 One-dimensional stationary description of scattering 287
11C.5 Integral equation for scattering problems 289
11C.6 The Born series 293
11C.7 Partial wave expansion 295
11C.8 s-Wave scattering states in the square-well potential 298
11C.9 Problems 299

12 Modern pictures of quantum mechanics 301
12.1 Quantum mechanics on phase space 301
12.2 Representations of the group algebra 304
12.3 Moyal brackets 308
12.4 Tomographic picture: preliminaries 309
12.5 Tomographic picture 311
12.5.1 Classical tomography 312
12.5.2 Quantum tomography 313
12.6 Pictures of quantum mechanics for a two-level system 315
12.6.1 von Neumann picture 317
12.6.2 Heisenberg picture 319
12.6.3 Unitary group $U(2)$ 320
12.6.4 A closer look at states in the Heisenberg picture 321
12.6.5 Weyl picture 322
12.6.6 Probability distributions and states 324
12.6.7 Ehrenfest picture 325
12.7 Composite systems 329
12.7.1 Inner product in tensor spaces 330
12.7.2 Complex linear operators in tensor spaces 330
12.7.3 Composite systems and Kronecker products 331
12.8 Identical particles 332
12.8.1 Product basis 332
12.8.2 Exchange symmetry 333
12.8.3 Exchange interaction 334
12.8.4 Two-electron atoms 335
12.9 Generalized paraFermi and paraBose oscillators 337
12.10 Problems 337

13 Formulations of quantum mechanics and their physical implications 339
13.1 Towards an overall view 339
13.2 From Schrödinger to Feynman 339
13.2.1 Remarks on the Feynman approach 341
13.3 Path integral for systems interacting with an electromagnetic field 344
13.4 Unification of quantum theory and special relativity 346
13.5 Dualities: quantum mechanics leads to new fundamental symmetries 351

14 Exam problems 353
14.1 End-of-year written exams 353

15 Definitions of geometric concepts 360
15.1 Outline 360
15.2 Groups 360
15.3 Lie groups 361
15.4 Symmetry 362
15.5 Various definitions of vector fields 363
15.6 Covariant vectors and 1-form fields 366
15.7 Lie algebras 368
15.8 Lie derivatives 369
15.9 Symplectic vector spaces 370
15.10 Homotopy maps and simply connected spaces 371
15.10.1 Examples of spaces which are or are not simply connected 372
15.11 Diffeomorphisms of manifolds 372
15.12 Foliations of manifolds 373

References 374
Index 381