

# Contents

**About the Authors** *ix*

**Preface** *xi*

<b>1</b>	<b>Preliminaries</b>	<b>1</b>
	Chapter Outline	1
	Chapter Objectives	1
1.1	From Statics	1
1.1.1	Mechanical Systems and Equilibrium Equations	1
1.1.2	Constraints and Free-Body Diagrams	1
1.1.3	Equilibrium Condition Via Virtual Work	2
1.2	From Kinematics	4
1.2.1	Kinematics of Particles	4
1.2.2	Kinematics of Rigid Bodies	5
1.2.2.1	Rigid Body in Translatory Motion	5
1.2.2.2	Rigid Body in Fixed-Axis Rotation	5
1.2.2.3	Rigid Body in General Plane Motion	6
1.2.3	Kinematics of Particles in Compound Motion	7
1.3	From Kinetics	8
1.3.1	Kinetics of Particles	8
1.3.2	Kinetics of Rigid Bodies	9
1.3.2.1	Kinetics of Rigid Bodies in Translatory Motion	9
1.3.2.2	Kinetics of Rigid Bodies in Fixed-Axis Rotation	10
1.3.2.3	Kinetics of Rigid Bodies in General Plane Motion	10
1.4	From Strength of Materials	13
1.4.1	Axial Loading	13
1.4.2	Torsion	14
1.4.3	Bending	14
<b>2</b>	<b>Lagrange's Equation for Mechanical Oscillatory Systems</b>	<b>17</b>
	Chapter Outline	17
	Chapter Objectives	17
2.1	About Lagrange's Equation of the Second Kind	17
2.2	Kinetic Energy in Mechanical Oscillatory Systems	19
2.3	Potential Energy in Mechanical Oscillatory Systems	21

2.3.1	Gravitational Potential Energy	22
2.3.2	Potential Energy of a Spring (Elastic Potential Energy)	24
2.3.2.1	On the Approximations for Linear Spring Deflection	25
2.4	Generalised Forces in Mechanical Oscillatory Systems	27
2.5	Dissipative Function in Mechanical Oscillatory Systems	28
	References	30
<b>3</b>	<b>Free Undamped Vibration of Single-Degree-of-Freedom Systems</b>	<b>31</b>
	Chapter Outline	31
	Chapter Objectives	31
	Theoretical Introduction	31
<b>4</b>	<b>Free Damped Vibration of Single-Degree-of-Freedom Systems</b>	<b>67</b>
	Chapter Outline	67
	Chapter Objectives	67
	Theoretical Introduction	67
<b>5</b>	<b>Forced Vibration of Single-Degree-of-Freedom Systems</b>	<b>101</b>
	Chapter Outline	101
	Chapter Objectives	101
	Theoretical Introduction	101
<b>6</b>	<b>Free Undamped Vibration of Two-Degree-of-Freedom Systems</b>	<b>127</b>
	Chapter Outline	127
	Chapter Objectives	127
	Theoretical Introduction	127
<b>7</b>	<b>Forced Vibration of Two-Degree-of-Freedom Systems</b>	<b>153</b>
	Chapter Outline	153
	Chapter Objectives	153
	Theoretical Introduction	153
<b>8</b>	<b>Vibration of Systems with Infinite Number of Degrees of Freedom</b>	<b>183</b>
	Chapter Outline	183
	Chapter Objectives	183
8.1	Theoretical Introduction: Longitudinal Vibration of Bars	183
8.2	Theoretical Introduction: Torsional Vibration of Shafts	197
8.3	Theoretical Introduction: Transversal Vibration of Beams	207
<b>9</b>	<b>Additional Topics</b>	<b>225</b>
	Chapter Outline	225
	Chapter Objectives	225
9.1	Theoretical Introduction	225
9.2	Equivalent Two-Element System for Concurrent Springs and Dampers	226
9.2.1	Concurrent Springs	227
9.2.2	Concurrent Dampers	231

9.3	Nonlinear Springs in Series	238
9.3.1	Purely Nonlinear Springs in Series	239
9.3.2	Equal Duffing Springs in Series	239
9.3.3	Two Different Nonlinear Springs	240
9.4	On the Deflection and Potential Energy of Nonlinear Springs: Approximate Expressions	242
9.4.1	Duffing-Type Spring Deformed in the Static Equilibrium Position	242
9.4.2	Duffing-Type Spring Undeformed in the Static Equilibrium Position	242
9.5	Corrections of Stiffness Properties of Certain Oscillatory Systems	244
9.5.1	One-Degree-of-Freedom Systems	245
9.5.1.1	Linear–Linear System	245
9.5.1.2	Duffing–Linear System	246
9.5.1.3	Duffing–Duffing System	248
9.5.2	Two-Degree-of-Freedom Systems	248
9.5.2.1	System with Two Pairs of Orthogonal Duffing Springs	248
9.5.2.2	System with Two Pairs of Equal and Orthogonal Duffing Springs	252
9.5.2.3	System with Two Pairs of Equal and Symmetrically Attached Duffing Springs	253
	<b>Appendix: Mathematical Topics</b>	255
A.1	Geometry	255
A.2	Trigonometry	257
A.3	Algebra	258
A.4	Vectors	258
A.5	Derivatives	259
A.6	Variation (Virtual Displacements)	260
A.7	Series	260
	<b>Index</b>	261