

Contents

<i>Preface</i>	xiii
<i>About the Author</i>	xix
<i>Supplementary Material</i>	xxi
Part 1: The Basics	1
1. Introduction	3
2. Application to Process Control	5
2.1 Benefit Estimation	5
2.2 Inferential Properties	7
2.3 Controller Performance Monitoring	7
2.4 Event Analysis	8
2.5 Time Series Analysis	9
3. Process Examples	11
3.1 Debutaniser	11
3.2 De-ethaniser	11
3.3 LPG Splitter	12
3.4 Propane Cargoes	17
3.5 Diesel Quality	17
3.6 Fuel Gas Heating Value	18
3.7 Stock Level	19
3.8 Batch Blending	22
4. Characteristics of Data	23
4.1 Data Types	23
4.2 Memory	24
4.3 Use of Historical Data	24
4.4 Central Value	25
4.5 Dispersion	32
4.6 Mode	33
4.7 Standard Deviation	35
4.8 Skewness and Kurtosis	37
4.9 Correlation	46
4.10 Data Conditioning	47
5. Probability Density Function	51
5.1 Uniform Distribution	55
5.2 Triangular Distribution	57
5.3 Normal Distribution	59
5.4 Bivariate Normal Distribution	62
5.5 Central Limit Theorem	65
5.6 Generating a Normal Distribution	69

5.7	Quantile Function	70
5.8	Location and Scale	71
5.9	Mixture Distribution	73
5.10	Combined Distribution	73
5.11	Compound Distribution	75
5.12	Generalised Distribution	75
5.13	Inverse Distribution	76
5.14	Transformed Distribution	76
5.15	Truncated Distribution	77
5.16	Rectified Distribution	78
5.17	Noncentral Distribution	78
5.18	Odds	79
5.19	Entropy	80
6.	Presenting the Data	83
6.1	Box and Whisker Diagram	83
6.2	Histogram	84
6.3	Kernel Density Estimation	90
6.4	Circular Plots	95
6.5	Parallel Coordinates	97
6.6	Pie Chart	98
6.7	Quantile Plot	98
7.	Sample Size	105
7.1	Mean	105
7.2	Standard Deviation	106
7.3	Skewness and Kurtosis	107
7.4	Dichotomous Data	108
7.5	Bootstrapping	110
8.	Significance Testing	113
8.1	Null Hypothesis	113
8.2	Confidence Interval	116
8.3	Six-Sigma	118
8.4	Outliers	119
8.5	Repeatability	120
8.6	Reproducibility	121
8.7	Accuracy	122
8.8	Instrumentation Error	123
9.	Fitting a Distribution	127
9.1	Accuracy of Mean and Standard Deviation	130
9.2	Fitting a CDF	131
9.3	Fitting a QF	134
9.4	Fitting a PDF	135
9.5	Fitting to a Histogram	138
9.6	Choice of Penalty Function	141
10.	Distribution of Dependent Variables	147
10.1	Addition and Subtraction	147
10.2	Division and Multiplication	148

10.3	Reciprocal	153
10.4	Logarithmic and Exponential Functions	153
10.5	Root Mean Square	162
10.6	Trigonometric Functions	164
11.	<i>Commonly Used Functions</i>	165
11.1	Euler's Number	165
11.2	Euler–Mascheroni Constant	166
11.3	Logit Function	166
11.4	Logistic Function	167
11.5	Gamma Function	168
11.6	Beta Function	174
11.7	Pochhammer Symbol	174
11.8	Bessel Function	176
11.9	Marcum Q-Function	178
11.10	Riemann Zeta Function	180
11.11	Harmonic Number	180
11.12	Stirling Approximation	182
11.13	Derivatives	183
12.	Selected Distributions	185
12.1	Lognormal	186
12.2	Burr	189
12.3	Beta	191
12.4	Hosking	195
12.5	Student t	204
12.6	Fisher	208
12.7	Exponential	210
12.8	Weibull	213
12.9	Chi-Squared	216
12.10	Gamma	221
12.11	Binomial	225
12.12	Poisson	231
13.	Extreme Value Analysis	235
14.	Hazard Function	245
15.	CUSUM	253
16.	Regression Analysis	259
16.1	F Test	275
16.2	Adjusted R^2	278
16.3	Akaike Information Criterion	279
16.4	Artificial Neural Networks	281
16.5	Performance Index	286
17.	Autocorrelation	291
18.	Data Reconciliation	299
19.	Fourier Transform	305

Part 2: Catalogue of Distributions	315
20. Normal Distribution	317
20.1 Skew-Normal	317
20.2 Gibrat	320
20.3 Power Lognormal	320
20.4 Logit-Normal	321
20.5 Folded Normal	321
20.6 Lévy	323
20.7 Inverse Gaussian	325
20.8 Generalised Inverse Gaussian	329
20.9 Normal Inverse Gaussian	330
20.10 Reciprocal Inverse Gaussian	332
20.11 Q-Gaussian	334
20.12 Generalised Normal	338
20.13 Exponentially Modified Gaussian	345
20.14 Moyal	347
21. Burr Distribution	349
21.1 Type I	349
21.2 Type II	349
21.3 Type III	349
21.4 Type IV	350
21.5 Type V	351
21.6 Type VI	351
21.7 Type VII	353
21.8 Type VIII	354
21.9 Type IX	354
21.10 Type X	355
21.11 Type XI	356
21.12 Type XII	356
21.13 Inverse	357
22. Logistic Distribution	361
22.1 Logistic	361
22.2 Half-Logistic	364
22.3 Skew-Logistic	365
22.4 Log-Logistic	367
22.5 Paralogistic	369
22.6 Inverse Paralogistic	370
22.7 Generalised Logistic	371
22.8 Generalised Log-Logistic	375
22.9 Exponentiated Kumaraswamy–Dagum	376
23. Pareto Distribution	377
23.1 Pareto Type I	377
23.2 Bounded Pareto Type I	378
23.3 Pareto Type II	379
23.4 Lomax	381
23.5 Inverse Pareto	381
23.6 Pareto Type III	382

23.7	Pareto Type IV	383
23.8	Generalised Pareto	383
23.9	Pareto Principle	385
24.	Stoppa Distribution	389
24.1	Type I	389
24.2	Type II	389
24.3	Type III	391
24.4	Type IV	391
24.5	Type V	392
25.	Beta Distribution	393
25.1	Arcsine	393
25.2	Wigner Semicircle	394
25.3	Balding–Nichols	395
25.4	Generalised Beta	396
25.5	Beta Type II	396
25.6	Generalised Beta Prime	399
25.7	Beta Type IV	400
25.8	PERT	401
25.9	Beta Rectangular	403
25.10	Kumaraswamy	404
25.11	Noncentral Beta	407
26.	Johnson Distribution	409
26.1	S_N	409
26.2	S_U	410
26.3	S_L	412
26.4	S_B	412
26.5	Summary	413
27.	Pearson Distribution	415
27.1	Type I	416
27.2	Type II	416
27.3	Type III	417
27.4	Type IV	418
27.5	Type V	424
27.6	Type VI	425
27.7	Type VII	429
27.8	Type VIII	433
27.9	Type IX	433
27.10	Type X	433
27.11	Type XI	434
27.12	Type XII	434
28.	Exponential Distribution	435
28.1	Generalised Exponential	435
28.2	Gompertz–Verhulst	435
28.3	Hyperexponential	436
28.4	Hypoexponential	437
28.5	Double Exponential	438

28.6	Inverse Exponential	439
28.7	Maxwell–Jüttner	439
28.8	Stretched Exponential	440
28.9	Exponential Logarithmic	441
28.10	Logistic Exponential	442
28.11	Q-Exponential	442
28.12	Benktander	445
29.	Weibull Distribution	447
29.1	Nukiyama–Tanasawa	447
29.2	Q-Weibull	447
30.	Chi Distribution	451
30.1	Half-Normal	451
30.2	Rayleigh	452
30.3	Inverse Rayleigh	454
30.4	Maxwell	454
30.5	Inverse Chi	458
30.6	Inverse Chi-Squared	459
30.7	Noncentral Chi-Squared	460
31.	Gamma Distribution	463
31.1	Inverse Gamma	463
31.2	Log-Gamma	463
31.3	Generalised Gamma	467
31.4	Q-Gamma	468
32.	Symmetrical Distributions	471
32.1	Anglit	471
32.2	Bates	472
32.3	Irwin–Hall	473
32.4	Hyperbolic Secant	475
32.5	Arctangent	476
32.6	Kappa	477
32.7	Laplace	478
32.8	Raised Cosine	479
32.9	Cardioid	481
32.10	Slash	481
32.11	Tukey Lambda	483
32.12	Von Mises	486
33.	Asymmetrical Distributions	487
33.1	Benini	487
33.2	Birnbaum–Saunders	488
33.3	Bradford	490
33.4	Champernowne	491
33.5	Davis	492
33.6	Fréchet	494
33.7	Gompertz	496
33.8	Shifted Gompertz	497
33.9	Gompertz–Makeham	498

33.10	Gamma-Gompertz	499
33.11	Hyperbolic	499
33.12	Asymmetric Laplace	502
33.13	Log-Laplace	504
33.14	Lindley	506
33.15	Lindley-Geometric	507
33.16	Generalised Lindley	509
33.17	Mielke	509
33.18	Muth	510
33.19	Nakagami	512
33.20	Power	513
33.21	Two-Sided Power	514
33.22	Exponential Power	516
33.23	Rician	517
33.24	Topp–Leone	517
33.25	Generalised Tukey Lambda	519
33.26	Wakeby	521
34.	Amoroso Distribution	525
35.	Binomial Distribution	529
35.1	Negative-Binomial	529
35.2	Pólya	531
35.3	Geometric	531
35.4	Beta-Geometric	535
35.5	Yule–Simon	536
35.6	Beta-Binomial	538
35.7	Beta-Negative Binomial	540
35.8	Beta-Pascal	541
35.9	Gamma-Poisson	542
35.10	Conway–Maxwell–Poisson	543
35.11	Skellam	546
36.	Other Discrete Distributions	549
36.1	Benford	549
36.2	Borel–Tanner	552
36.3	Consul	555
36.4	Delaporte	556
36.5	Flory–Schulz	558
36.6	Hypergeometric	559
36.7	Negative Hypergeometric	561
36.8	Logarithmic	561
36.9	Discrete Weibull	563
36.10	Zeta	564
36.11	Zipf	565
36.12	Parabolic Fractal	567
<i>Appendix 1</i>	<i>Data Used in Examples</i>	569
<i>Appendix 2</i>	<i>Summary of Distributions</i>	577
<i>References</i>		591
<i>Index</i>		593