

Contents

Preface	xv
1 Introduction	1
1.1 Background	1
1.2 Scope	1
1.3 Notation	6
1.4 Distributions related to the Normal distribution	8
1.4.1 Normal distributions	8
1.4.2 Chi-squared distribution	9
1.4.3 t-distribution	10
1.4.4 F-distribution	10
1.4.5 Some relationships between distributions	11
1.5 Quadratic forms	11
1.6 Estimation	13
1.6.1 Maximum likelihood estimation	13
1.6.2 Example: Poisson distribution	15
1.6.3 Least squares estimation	15
1.6.4 Comments on estimation	16
1.6.5 Example: Tropical cyclones	17
1.7 Exercises	17
2 Model Fitting	21
2.1 Introduction	21
2.2 Examples	21
2.2.1 Chronic medical conditions	21
2.2.2 Example: Birthweight and gestational age	25
2.3 Some principles of statistical modelling	35
2.3.1 Exploratory data analysis	35
2.3.2 Model formulation	36
2.3.3 Parameter estimation	36

2.3.4	Residuals and model checking	36
2.3.5	Inference and interpretation	39
2.3.6	Further reading	40
2.4	Notation and coding for explanatory variables	40
2.4.1	Example: Means for two groups	41
2.4.2	Example: Simple linear regression for two groups	42
2.4.3	Example: Alternative formulations for comparing the means of two groups	42
2.4.4	Example: Ordinal explanatory variables	43
2.5	Exercises	44
3	Exponential Family and Generalized Linear Models	49
3.1	Introduction	49
3.2	Exponential family of distributions	50
3.2.1	Poisson distribution	51
3.2.2	Normal distribution	52
3.2.3	Binomial distribution	52
3.3	Properties of distributions in the exponential family	53
3.4	Generalized linear models	56
3.5	Examples	58
3.5.1	Normal linear model	58
3.5.2	Historical linguistics	58
3.5.3	Mortality rates	59
3.6	Exercises	61
4	Estimation	65
4.1	Introduction	65
4.2	Example: Failure times for pressure vessels	65
4.3	Maximum likelihood estimation	70
4.4	Poisson regression example	73
4.5	Exercises	76
5	Inference	79
5.1	Introduction	79
5.2	Sampling distribution for score statistics	81
5.2.1	Example: Score statistic for the Normal distribution	82
5.2.2	Example: Score statistic for the Binomial distribution	82
5.3	Taylor series approximations	83
5.4	Sampling distribution for maximum likelihood estimators	84

5.4.1	Example: Maximum likelihood estimators for the Normal linear model	85
5.5	Log-likelihood ratio statistic	86
5.6	Sampling distribution for the deviance	87
5.6.1	Example: Deviance for a Binomial model	88
5.6.2	Example: Deviance for a Normal linear model	89
5.6.3	Example: Deviance for a Poisson model	91
5.7	Hypothesis testing	92
5.7.1	Example: Hypothesis testing for a Normal linear model	94
5.8	Exercises	95
6	Normal Linear Models	97
6.1	Introduction	97
6.2	Basic results	98
6.2.1	Maximum likelihood estimation	98
6.2.2	Least squares estimation	98
6.2.3	Deviance	99
6.2.4	Hypothesis testing	99
6.2.5	Orthogonality	100
6.2.6	Residuals	101
6.2.7	Other diagnostics	102
6.3	Multiple linear regression	104
6.3.1	Example: Carbohydrate diet	104
6.3.2	Coefficient of determination, R^2	108
6.3.3	Model selection	111
6.3.4	Collinearity	118
6.4	Analysis of variance	119
6.4.1	One-factor analysis of variance	119
6.4.2	Two-factor analysis of variance	126
6.5	Analysis of covariance	132
6.6	General linear models	135
6.7	Non-linear associations	137
6.7.1	<i>PLOS Medicine</i> journal data	138
6.8	Fractional polynomials	141
6.9	Exercises	143

7 Binary Variables and Logistic Regression	149
7.1 Probability distributions	149
7.2 Generalized linear models	150
7.3 Dose response models	151
7.3.1 Example: Beetle mortality	154
7.4 General logistic regression model	158
7.4.1 Example: Embryogenic anthers	159
7.5 Goodness of fit statistics	162
7.6 Residuals	166
7.7 Other diagnostics	167
7.8 Example: Senility and WAIS	168
7.9 Odds ratios and prevalence ratios	171
7.10 Exercises	174
8 Nominal and Ordinal Logistic Regression	179
8.1 Introduction	179
8.2 Multinomial distribution	180
8.3 Nominal logistic regression	181
8.3.1 Example: Car preferences	183
8.4 Ordinal logistic regression	188
8.4.1 Cumulative logit model	189
8.4.2 Proportional odds model	189
8.4.3 Adjacent categories logit model	190
8.4.4 Continuation ratio logit model	191
8.4.5 Comments	192
8.4.6 Example: Car preferences	192
8.5 General comments	193
8.6 Exercises	194
9 Poisson Regression and Log-Linear Models	197
9.1 Introduction	197
9.2 Poisson regression	198
9.2.1 Example of Poisson regression: British doctors' smoking and coronary death	201
9.3 Examples of contingency tables	204
9.3.1 Example: Cross-sectional study of malignant melanoma	205
9.3.2 Example: Randomized controlled trial of influenza vaccine	206

9.3.3 Example: Case-control study of gastric and duodenal ulcers and aspirin use	207
9.4 Probability models for contingency tables	209
9.4.1 Poisson model	209
9.4.2 Multinomial model	209
9.4.3 Product multinomial models	210
9.5 Log-linear models	210
9.6 Inference for log-linear models	212
9.7 Numerical examples	212
9.7.1 Cross-sectional study of malignant melanoma	212
9.7.2 Case-control study of gastric and duodenal ulcer and aspirin use	215
9.8 Remarks	216
9.9 Exercises	217
10 Survival Analysis	223
10.1 Introduction	223
10.2 Survivor functions and hazard functions	225
10.2.1 Exponential distribution	226
10.2.2 Proportional hazards models	227
10.2.3 Weibull distribution	228
10.3 Empirical survivor function	230
10.3.1 Example: Remission times	231
10.4 Estimation	233
10.4.1 Example: Exponential model	234
10.4.2 Example: Weibull model	235
10.5 Inference	236
10.6 Model checking	236
10.7 Example: Remission times	238
10.8 Exercises	240
11 Clustered and Longitudinal Data	245
11.1 Introduction	245
11.2 Example: Recovery from stroke	247
11.3 Repeated measures models for Normal data	253
11.4 Repeated measures models for non-Normal data	257
11.5 Multilevel models	259
11.6 Stroke example continued	262
11.7 Comments	265
11.8 Exercises	266

12 Bayesian Analysis	271
12.1 Frequentist and Bayesian paradigms	271
12.1.1 Alternative definitions of p-values and confidence intervals	271
12.1.2 Bayes' equation	272
12.1.3 Parameter space	273
12.1.4 Example: Schistosoma japonicum	273
12.2 Priors	275
12.2.1 Informative priors	276
12.2.2 Example: Sceptical prior	276
12.2.3 Example: Overdoses amongst released prisoners	279
12.3 Distributions and hierarchies in Bayesian analysis	281
12.4 WinBUGS software for Bayesian analysis	281
12.5 Exercises	284
13 Markov Chain Monte Carlo Methods	287
13.1 Why standard inference fails	287
13.2 Monte Carlo integration	287
13.3 Markov chains	289
13.3.1 The Metropolis–Hastings sampler	291
13.3.2 The Gibbs sampler	293
13.3.3 Comparing a Markov chain to classical maximum likelihood estimation	295
13.3.4 Importance of parameterization	299
13.4 Bayesian inference	300
13.5 Diagnostics of chain convergence	302
13.5.1 Chain history	302
13.5.2 Chain autocorrelation	304
13.5.3 Multiple chains	305
13.6 Bayesian model fit: the deviance information criterion	306
13.7 Exercises	308
14 Example Bayesian Analyses	315
14.1 Introduction	315
14.2 Binary variables and logistic regression	316
14.2.1 Prevalence ratios for logistic regression	319
14.3 Nominal logistic regression	322
14.4 Latent variable model	324
14.5 Survival analysis	326
14.6 Random effects	328

14.7 Longitudinal data analysis	331
14.8 Bayesian model averaging	338
14.8.1 Example: Stroke recovery	340
14.8.2 Example: <i>PLOS Medicine</i> journal data	340
14.9 Some practical tips for WinBUGS	342
14.10 Exercises	344
Postface	347
Appendix	355
Software	357
References	359
Index	371