Contents

Preface xv

1. Matrices 1
1.1 Matrix Arithmetic 1
1.1.1 Matrix Arithmetic 3
1.1.1.1 Matrix Addition 3
1.1.1.2 Scalar Multiplication 3
1.1.1.3 Matrix Multiplication 3
Exercises 7
1.2 The Algebra of Matrices 9
1.2.1 Properties of Matrix Addition, Scalar Multiplication, and Matrix Multiplication 9
1.2.2 The Identity Matrix 12
1.2.3 The Inverse of a Square Matrix 12
1.2.4 Determinants 14
1.2.5 Elementary Matrices 17
1.2.6 Matrices That Interchange Two Rows of a Matrix 18
1.2.7 Multiplying a Row of a Matrix by a Constant 18
1.2.8 Adding a Multiple of One Row to Another Row 19
1.2.9 Computing the Inverse of a Matrix 19
1.2.10 The Transpose of a Matrix 22
Exercises 23
1.3 The *LU* Decomposition of a Square Matrix (Optional) 29
Exercises 32

2. Systems of Linear Equations 33
2.1 Basic Definitions 33
Exercises 34
2.2 Solving Systems of Linear Equations (Gaussian Elimination) 34
2.2.1 Solving Systems of Linear Equations 36
2.2.2 Using Technology to Accomplish Gaussian Elimination 40
Exercises 40
2.3 Equivalent Systems of Linear Equations 41
2.3.1 Row Reduced Form of a Matrix 43
Exercises 44
2.4 Expressing the Solution of a System of Linear Equations 44
2.4.1 Systems of Linear Equations That Have No Solutions 44
2.4.2 Systems of Linear Equations That Have Exactly One Solution 45
2.4.3 Systems of Linear Equations That Have Infinitely Many Solutions 45
2.4.4 Application of Linear Systems to Curve Fitting 47
Exercises 51
2.5 Expressing Systems of Linear Equations in Other Forms 53
2.5.1 Representing a System of Linear Equations as a Vector Equation 53
2.5.2 Equivalence of a System of Linear Equations and a Matrix Equation 55
Exercises 58

2.6 Applications 60
2.6.1 Flow Problems 60
2.6.2 Example: Kirchoff's Laws 61
2.6.3 Balancing Chemical Equations Using Linear Algebra 63
Exercises 64
2.6.4 Markov Chains 65
Exercises 71

3. Vector Spaces 73
3.1 Vector Spaces in $\mathbb{R}^n$ 73
Exercises 80
3.2 Axioms and Examples of Vector Spaces 81
3.2.1 Some Examples of Sets That Are Not Vector Spaces 83
3.2.2 Additional Properties of Vector Spaces 84
Exercises 86
3.3 Subspaces of a Vector Space 87
Exercises 91
3.4 Spanning Sets, Linearly Independent Sets and Bases 92
Exercises 101
3.5 Converting a Set of Vectors to a Basis 103
3.6 A Synopsis of Sections 3.3.4 and 3.3.5 108
Exercises 109
3.7 Change of Bases 110
Exercises 117
3.8 Null Space, Row Space, and Column Space of a Matrix 118
Exercises 128
3.9 Sums and Direct Sums of Vector Spaces (Optional) 130
Exercises 134

4. Linear Transformations 137
4.1 Properties of a Linear Transformation 137
4.1.1 Null Space and Range (Image) of a Linear Transformation 143
Exercises 144
4.2 Representing a Linear Transformation 146
4.2.1 Representation of a Linear Transformation in the Usual Basis 147
Exercises 152
4.3 Finding the Representation of a Linear Operator with respect to Different Bases 154
Exercises 156
4.4 Composition (Multiplication) of Linear Transformations 158
Exercises 160

5. Eigenvalues and Eigenvectors 163
5.1 Determining Eigenvalues and Eigenvectors 163
5.1.1 Finding the Eigenvectors after the Eigenvalues Have Been Found 165
Exercises 170

5.2 Diagonalizing a Matrix 172
5.2.1 Algebraic and Geometric Multiplicities of an Eigenvalue 173
5.2.2 Diagonalizing a Matrix 173
Exercises 176
5.3 Similar Matrices 177
Exercises 182
5.4 Eigenvalues and Eigenvectors in Systems of Differential Equations 183
Exercises 188

6. Inner Product Spaces 191
6.1 Some Facts about Complex Numbers 191
Exercises 192
6.2 Inner Product Spaces 194
Exercises 196
6.3 Orthogonality 197
Exercises 200
6.4 The Gram–Schmidt Process 201
6.4.1 Algorithm for the Gram–Schmidt Process 201
Exercises 205
6.5 Representation of a Linear Transformation on Inner Product Spaces (Optional) 207
Exercises 209
6.6 Orthogonal Complement 210
Exercises 212
6.7 Four Subspaces Associated with a Matrix (Optional) 213
6.8 Projections 214
Exercises 221
6.9 Least-Squares Estimates in Statistics (Optional) 222
Exercise 225
6.10 Weighted Inner Products (Optional) 225
Reference 226

7. Linear Functionals, Dual Spaces, and Adjoint Operators 227
7.1 Linear Functionals 227
7.1.1 The Second Dual of a Vector Space (Optional) 231
Exercises 232
7.2 The Adjoint of a Linear Operator 233
7.2.1 The Adjoint Operator 233
7.2.2 Adjoint on Weighted Inner Product Spaces (Optional) 239
Exercises 240
7.3 The Spectral Theorem 242
Exercises 248

Appendix A: A Brief Guide to MATLAB® 249

Appendix B: An Introduction to R 263

Appendix C: Downloading R to Your Computer 279

Answers to Selected Exercises 281

Index 301