



NONLINEAR PHENOMENA IN POWER ELECTRONICS

ATTRACTORS, BIFURCATIONS,
CHAOS, AND NONLINEAR
CONTROL



Edited by
Soumitro Banerjee
George C. Verghese

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