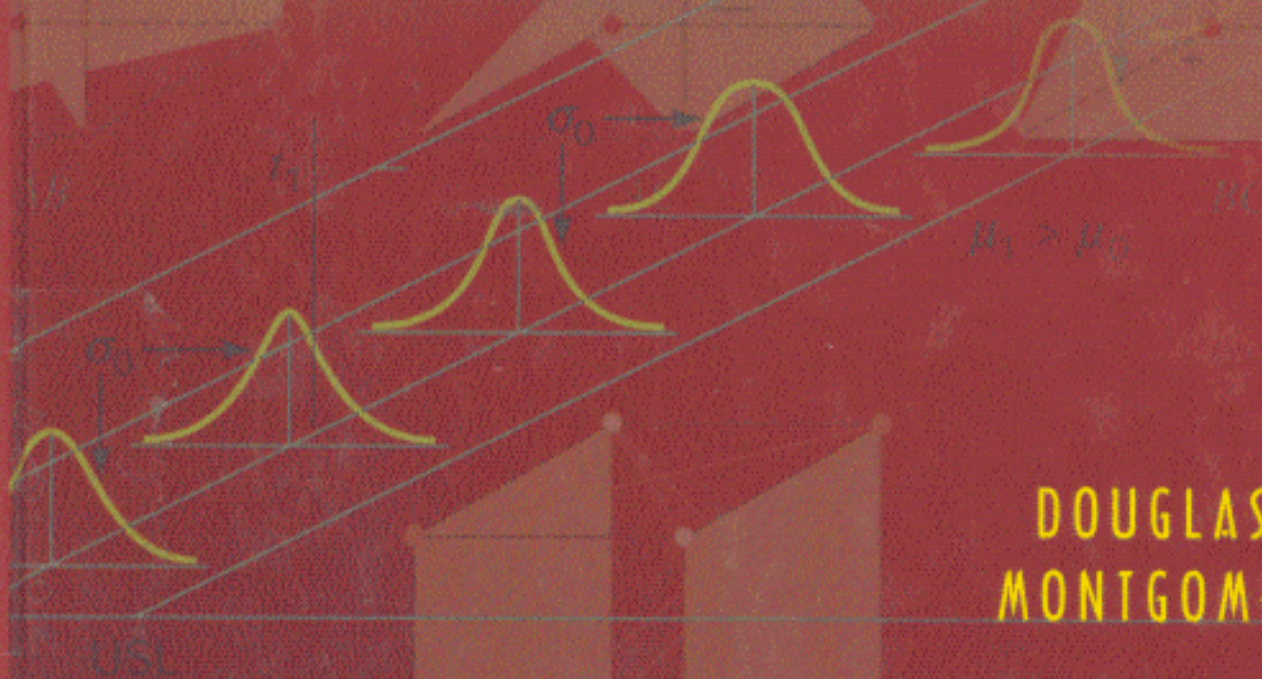


4<sup>TH</sup> EDITION

# INTRODUCTION TO STATISTICAL QUALITY CONTROL



DOUGLAS C.  
MONTGOMERY

# Contents

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## CHAPTER 1

### Quality Improvement in the Modern Business Environment 1

- 1-1 The Meaning of Quality and Quality Improvement 2
  - 1-1.1 Dimensions of Quality 2
  - 1-1.2 Quality Engineering Terminology 6
- 1-2 A Brief History of Quality Control and Improvement 8
- 1-3 Statistical Methods for Quality Control and Improvement 12
- 1-4 Other Aspects of Quality Control and Improvement 17
  - 1-4.1 Quality Philosophy and Management Strategies 18
  - 1-4.2 The Link between Quality and Productivity 25
  - 1-4.3 Quality Costs 26
  - 1-4.4 Legal Aspects of Quality 32
  - 1-4.5 Implementing Quality Improvement 34

---

## PART I

### Statistical Methods Useful in Quality Improvement 37

---

## CHAPTER 2

### Modeling Process Quality 39

- 2-1 Describing Variation 40
  - 2-1.1 The Stem-and-Leaf Plot 40

- 2-1.2 The Frequency Distribution and Histogram 43
- 2-1.3 Numerical Summary of Data 45
- 2-1.4 The Box Plot 48
- 2-1.5 Sample Computer Output 49
- 2-1.6 Probability Distributions 51
- 2-2 Important Discrete Distributions 56
  - 2-2.1 The Hypergeometric Distribution 56
  - 2-2.2 The Binomial Distribution 58
  - 2-2.3 The Poisson Distribution 60
  - 2-2.4 The Pascal and Related Distributions 61
- 2-3 Important Continuous Distributions 62
  - 2-3.1 The Normal Distribution 63
  - 2-3.2 The Exponential Distribution 69
  - 2-3.3 The Gamma Distribution 71
  - 2-3.4 The Weibull Distribution 73
- 2-4 Some Useful Approximations 74
  - 2-4.1 The Binomial Approximation to the Hypergeometric 75
  - 2-4.2 The Poisson Approximation to the Binomial 75
  - 2-4.3 The Normal Approximation to the Binomial 75
  - 2-4.4 Comments on Approximations 76
- 2-5 Exercises 77

## CHAPTER 3

### Inferences about Process Quality 82

- 3-1 Statistics and Sampling Distributions 83
  - 3-1.1 Sampling from a Normal Distribution 85

- 3-1.2 Sampling from a Bernoulli Distribution 89
- 3-1.3 Sampling from a Poisson Distribution 89
- 3-2 Point Estimation of Process Parameters 90
- 3-3 Statistical Inference for a Single Sample 93
  - 3-3.1 Inference on the Mean of a Population, Variance Known 94
  - 3-3.2 The Use of *P*-Values for Hypothesis Testing 98
  - 3-3.3 Inference on the Mean of a Normal Distribution, Variance Unknown 99
  - 3-3.4 Inference on the Variance of a Normal Distribution 103
  - 3-3.5 Inference on a Population Proportion 105
  - 3-3.6 The Probability of Type II Error 107
  - 3-3.7 Probability Plotting 110
- 3-4 Statistical Inference for Two Samples 114
  - 3-4.1 Inference for a Difference in Means, Variances Known 114
  - 3-4.2 Inference for a Difference in Means of Two Normal Distributions, Variances Unknown 117
  - 3-4.3 Inference on the Variances of Two Normal Distributions 127
  - 3-4.4 Inference on Two Population Proportions 128
- 3-5 What If There Are More Than Two Populations? The Analysis of Variance 130
  - 3-5.1 An Example 131
  - 3-5.2 The Analysis of Variance 133
  - 3-5.3 Checking Assumptions: Residual Analysis 140
- 3-6 Exercises 142

---

## PART II

### Basic Methods of Statistical Process Control and Capability Analysis 151

---

#### CHAPTER 4

#### *Methods and Philosophy of Statistical Process Control* 153

- 4-1 Introduction 154
- 4-2 Chance and Assignable Causes of Quality Variation 154
- 4-3 Statistical Basis of the Control Chart 156
  - 4-3.1 Basic Principles 156
  - 4-3.2 Choice of Control Limits 164
  - 4-3.3 Sample Size and Sampling Frequency 166
  - 4-3.4 Rational Subgroups 170
  - 4-3.5 Analysis of Patterns on Control Charts 172
  - 4-3.6 Discussion of Sensitizing Rules for Control Charts 175
- 4-4 The Rest of the "Magnificent Seven" 177
- 4-5 Implementing SPC 184
- 4-6 An Application of SPC 186
- 4-7 Nonmanufacturing Applications of Statistical Process Control 193
- 4-8 Exercises 201

#### CHAPTER 5

#### Control Charts for Variables 206

- 5-1 Introduction 207
- 5-2 Control Charts for  $\bar{x}$  and *R* 207
  - 5-2.1 Statistical Basis of the Charts 207
  - 5-2.2 Development and Use of  $\bar{x}$  and *R* Charts 212
  - 5-2.3 Charts Based on Standard Values 228
  - 5-2.4 Interpretation of  $\bar{x}$  and *R* Charts 229
  - 5-2.5 The Effect of Nonnormality on  $\bar{x}$  and *R* Charts 232
  - 5-2.6 The Operating-Characteristic Function 233

- 5-2.7 The Average Run Length for the  $\bar{x}$  Chart **236**
- 5-3 Control Charts for  $\bar{x}$  and  $S$  **239**
  - 5-3.1 Construction and Operation of  $\bar{x}$  and  $S$  Charts **239**
  - 5-3.2 The  $\bar{x}$  and  $S$  Control Charts with Variable Sample Size **244**
  - 5-3.3 The  $S^2$  Control Chart **248**
- 5-4 The Shewhart Control Chart for Individual Measurements **249**
- 5-5 Summary of Procedures for  $\bar{x}$ ,  $R$ , and  $S$  Charts **260**
- 5-6 Applications of Variables Control Charts **260**
- 5-7 Exercises **265**

## CHAPTER 6

### Control Charts for Attributes **283**

- 6-1 Introduction **284**
- 6-2 The Control Chart for Fraction Nonconforming **284**
  - 6-2.1 Development and Operation of the Control Chart **286**
  - 6-2.2 Variable Sample Size **298**
  - 6-2.3 Nonmanufacturing Applications **303**
  - 6-2.4 The Operating-Characteristic Function and Average Run Length Calculations **305**
- 6-3 Control Charts for Nonconformities (Defects) **308**
  - 6-3.1 Procedures with Constant Sample Size **308**
  - 6-3.2 Procedures with Variable Sample Size **319**
  - 6-3.3 Demerit Systems **322**
  - 6-3.4 The Operating-Characteristic Function **324**
  - 6-3.5 Dealing with Low Defect Levels **325**
  - 6-3.6 Nonmanufacturing Applications **328**
- 6-4 Choice between Attributes and Variables Control Charts **329**

- 6-5 Guidelines for Implementing Control Charts **333**

- 6-6 Exercises **339**

## CHAPTER 7

### Process and Measurement System Capability Analysis **349**

- 7-1 Introduction **350**
- 7-2 Process Capability Analysis Using a Histogram or a Probability Plot **352**
  - 7-2.1 Using the Histogram **352**
  - 7-2.2 Probability Plotting **355**
- 7-3 Process Capability Ratios **357**
  - 7-3.1 Use and Interpretation of  $C_p$  **357**
  - 7-3.2 Process Capability Ratio for an Off-Center Process **362**
  - 7-3.3 Normality and the Process Capability Ratio **364**
  - 7-3.4 More about Process Centering **365**
  - 7-3.5 Confidence Intervals and Tests on Process Capability Ratios **367**
- 7-4 Process Capability Analysis Using a Control Chart **373**
- 7-5 Process Capability Analysis Using Designed Experiments **376**
- 7-6 Gage and Measurement System Capability Studies **377**
  - 7-6.1 Control Charts and Tabular Methods **377**
  - 7-6.2 Methods Based on Analysis of Variance **384**
- 7-7 Setting Specification Limits on Discrete Components **388**
  - 7-7.1 Linear Combinations **388**
  - 7-7.2 Nonlinear Combinations **392**
- 7-8 Estimating the Natural Tolerance Limits of a Process **394**
  - 7-8.1 Tolerance Limits Based on the Normal Distribution **395**
  - 7-8.2 Nonparametric Tolerance Limits **396**
- 7-9 Exercises **397**

**PART III****Other Statistical Process Monitoring and Control Techniques 403****CHAPTER 8****Cumulative Sum and Exponentially Weighted Moving Average Control Charts 405**

- 8-1 The Cumulative Sum Control Chart **406**
  - 8-1.1 Basic Principles: The Cusum Control Chart for Monitoring the Process Mean **406**
  - 8-1.2 The Tabular or Algorithmic Cusum for Monitoring the Process Mean **410**
  - 8-1.3 Recommendations for Cusum Design **415**
  - 8-1.4 The Standardized Cusum **417**
  - 8-1.5 Rational Subgroups **418**
  - 8-1.6 Improving Cusum Responsiveness for Large Shifts **418**
  - 8-1.7 The Fast Initial Response or Headstart Feature **419**
  - 8-1.8 One-Sided Cusums **421**
  - 8-1.9 A Cusum for Monitoring Process Variability **421**
  - 8-1.10 Cusums for Other Sample Statistics **422**
  - 8-1.11 The V-Mask Procedure **423**
- 8-2 The Exponentially Weighted Moving Average Control Chart **425**
  - 8-2.1 The Exponentially Weighted Moving Average Control Chart for Monitoring the Process Mean **426**
  - 8-2.2 Design of an EWMA Control Chart **431**
  - 8-2.3 Rational Subgroups **432**
  - 8-2.4 Robustness of the EWMA to Nonnormality **432**
  - 8-2.5 Extensions of the EWMA **433**
- 8-3 The Moving Average Control Chart **437**
- 8-4 Exercises **440**

**CHAPTER 9****Other Univariate Statistical Process Monitoring and Control Techniques 443**

- 9-1 Statistical Process Control for Short Production Runs **444**
  - 9-1.1  $\bar{x}$  and  $R$  Charts for Short Production Runs **444**
  - 9-1.2 Attributes Control Charts for Short Production Runs **447**
  - 9-1.3 Other Methods **448**
- 9-2 Modified and Acceptance Control Charts **449**
  - 9-2.1 Modified Control Limits for the  $\bar{x}$  Chart **449**
  - 9-2.2 Acceptance Control Charts **453**
- 9-3 Control Charts for Multiple-Stream Processes **454**
  - 9-3.1 Multiple-Stream Processes **454**
  - 9-3.2 Group Control Charts **455**
  - 9-3.3 Other Approaches **457**
- 9-4 SPC with Autocorrelated Process Data **458**
  - 9-4.1 Sources and Effects of Autocorrelation in Process Data **458**
  - 9-4.2 Model-Based Approaches **462**
  - 9-4.3 A Model-Free Approach **473**
- 9-5 Adaptive Sampling Procedures **478**
- 9-6 Economic Design of Control Charts **479**
  - 9-6.1 Designing a Control Chart **479**
  - 9-6.2 Process Characteristics **480**
  - 9-6.3 Cost Parameters **481**
  - 9-6.4 Early Work and Semieconomic Designs **482**
  - 9-6.5 An Economic Model of the  $\bar{x}$  Control Chart **484**
  - 9-6.6 Other Work **493**
- 9-7 Overview of Other Procedures **494**
  - 9-7.1 Tool Wear **494**
  - 9-7.2 Control Charts Based on Other Sample Statistics **495**
  - 9-7.3 Fill Control Problems **496**

- 9-7.4 Precontrol 497  
 9-8 Exercises 499

## CHAPTER 10

### Multivariate Process Monitoring and Control 507

- 10-1 The Multivariate Quality Control Problem 508  
 10-2 Description of Multivariate Data 510  
   10-2.1 The Multivariate Normal Distribution 510  
   10-2.2 The Sample Mean Vector and Covariance Matrix 511  
 10-3 The Hotelling  $T^2$  Control Chart 512  
   10-3.1 Subgrouped Data 512  
   10-3.2 Individual Observations 522  
 10-4 The Multivariate EWMA Control Chart 526  
 10-5 Regression Adjustment 530  
 10-6 Control Charts for Monitoring Variability 532  
 10-7 Latent Structure Methods 535  
   10-7.1 Principal Components 535  
   10-7.2 Partial Least Squares 542  
 10-8 Exercises 542

## CHAPTER 11

### Engineering Process Control and SPC 546

- 11-1 Process Monitoring and Process Regulation 547  
 11-2 Process Control by Feedback Adjustment 548  
   11-2.1 A Simple Adjustment Scheme: Integral Control 548  
   11-2.2 The Adjustment Chart 555  
   11-2.3 Variations of the Adjustment Chart 557  
   11-2.4 Other Types of Feedback Controllers 561  
 11-3 Combining SPC and EPC 562  
 11-4 Exercises 566

---

## PART IV

### Process Design and Improvement with Designed Experiments 569

---

## CHAPTER 12

### Factorial and Fractional Factorial Experiments for Process Design and Improvement 571

- 12-1 What Is Experimental Design? 572  
 12-2 Examples of Designed Experiments in Process Improvement 573  
 12-3 Guidelines for Designing Experiments 577  
 12-4 Factorial Experiments 579  
   12-4.1 An Example 582  
   12-4.2 Statistical Analysis 583  
   12-4.3 Residual Analysis 588  
 12-5 The  $2^k$  Factorial Design 591  
   12-5.1 The  $2^2$  Design 591  
   12-5.2 The  $2^k$  Design for  $k \geq 3$  Factors 598  
   12-5.3 A Single Replicate of the  $2^k$  Design 611  
   12-5.4 Addition of Center Points to the  $2^k$  Design 615  
   12-5.5 Blocking and Confounding in the  $2^k$  Design 620  
 12-6 Fractional Replication of the  $2^k$  Design 622  
   12-6.1 The One-Half Fraction of the  $2^k$  Design 622  
   12-6.2 Smaller Fractions: The  $2^{k-p}$  Fractional Factorial Design 628  
 12-7 Exercises 635

## CHAPTER 13

### Process Optimization with Designed Experiments 639

- 13-1 Response Surface Methods and Designs 640  
   13-1.1 The Method of Steepest Ascent 642  
   13-1.2 Analysis of a Second-Order Response Surface 645  
 13-2 Process Robustness Studies 651  
   13-2.1 Background 651

13-2.2 The Response Surface Approach  
to Process Robustness  
Studies **653**

13-3 Evolutionary Operation **662**

13-4 Exercises **669**

14-5.2 LTPD Plans **716**

14-5.3 Estimation of Process Average **719**

14-6 Exercises **719**

---

**PART V**  
**Acceptance Sampling 673**

---

**CHAPTER 14**

**Lot-by-Lot Acceptance Sampling for  
Attributes 675**

14-1 The Acceptance-Sampling Problem **676**

14-1.1 Advantages and Disadvantages of  
Sampling **677**

14-1.2 Types of Sampling Plans **678**

14-1.3 Lot Formation **679**

14-1.4 Random Sampling **680**

14-1.5 Guidelines for Using Acceptance  
Sampling **680**

14-2 Single-Sampling Plans for Attributes **682**

14-2.1 Definition of a Single-Sampling  
Plan **682**

14-2.2 The OC Curve **682**

14-2.3 Designing a Single-Sampling Plan  
with a Specified OC Curve **689**

14-2.4 Rectifying Inspection **690**

14-3 Double, Multiple, and Sequential  
Sampling **694**

14-3.1 Double-Sampling Plans **694**

14-3.2 Multiple-Sampling Plans **701**

14-3.3 Sequential-Sampling Plans **701**

14-4 Military Standard 105E (ANSI/ASQC Z1.4,  
ISO 2859) **705**

14-4.1 Description of the Standard **705**

14-4.2 Procedure **707**

14-4.3 Discussion **712**

14-5 The Dodge-Romig Sampling  
Plans **714**

14-5.1 AOQL Plans **716**

**CHAPTER 15**

**Other Acceptance-Sampling Techniques 722**

15-1 Acceptance Sampling by Variables **723**

15-1.1 Advantages and Disadvantages of  
Variables Sampling **723**

15-1.2 Types of Sampling Plans  
Available **724**

15-1.3 Caution in the Use of Variables  
Sampling **725**

15-2 Designing a Variables Sampling Plan with a  
Specified OC Curve **726**

15-3 MIL STD 414 (ANSI/ASQC Z1.9) **729**

15-3.1 General Description of the  
Standard **729**

15-3.2 Use of the Tables **730**

15-3.3 Discussion of MIL STD 414 and  
ANSI/ASQC Z1.9 **733**

15-4 Other Variables Sampling Procedures **734**

15-4.1 Sampling by Variables to Give Assurance  
Regarding the Lot or  
Process Mean **734**

15-4.2 Sequential Sampling by  
Variables **735**

15-5 Chain Sampling **735**

15-6 Continuous Sampling **737**

15-6.1 CSP-1 **738**

15-6.2 Other Continuous-Sampling  
Plans **740**

15-7 Skip-Lot Sampling Plans **741**

15-8 Exercises **745**

**Appendix 749**

I. Summary of Common Probability  
Distributions Often Used in Statistical  
Quality Control **751**

II. Cumulative Standard Normal Distribution **752**

III. Percentage Points of the  $\chi^2$  Distribution **754**

IV. Percentage Points of the $t$ Distribution	<b>755</b>
V. Percentage Points of the $F$ Distribution	<b>756</b>
VI. Factors for Constructing Variables Control Charts	<b>761</b>
VII. Factors for Two-Sided Normal Tolerance Limits	<b>762</b>
VIII. Factors for One-Sided Normal Tolerance Limits	<b>763</b>

IX. Random Numbers	<b>764</b>
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Bibliography	<b>765</b>
--------------	------------

Answers to Selected Exercises	<b>777</b>
-------------------------------	------------

Index	<b>791</b>
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